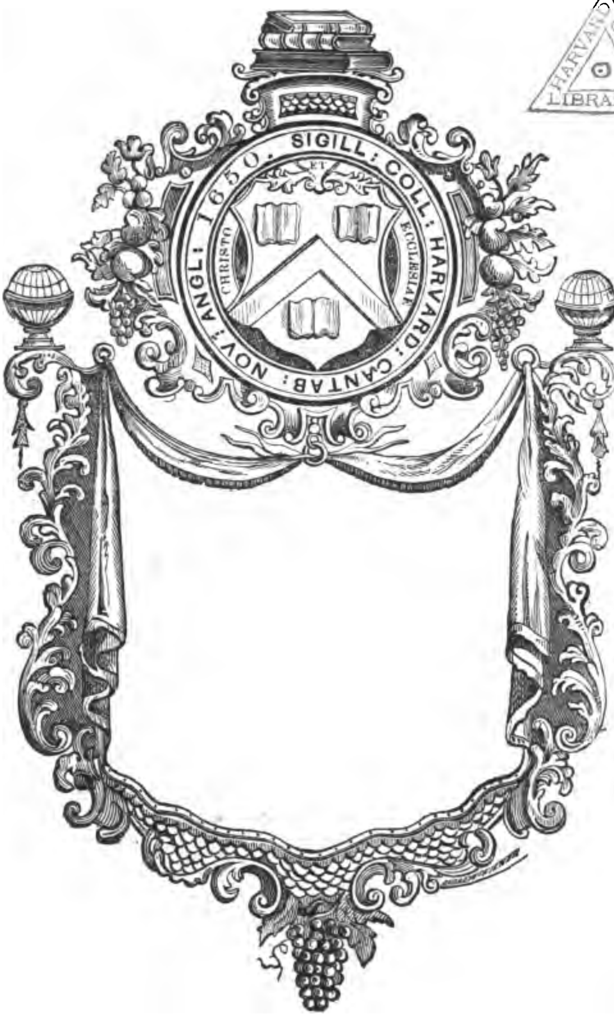
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CHRISTO ET ECCLESIAE

ARITHMETIC;

AS TAUGHT IN THE

TROY EPISCOPAL INSTITUTE;

BY W. F. WALKER, A. M., PRINCIPAL.

Out of the olde fieldes, as men saithe,
  Cometh all this newe corne fro yere to yere;
And out of olde bookes, in good faithe,
  Cometh all this newe science that men lere.

Chaucer.

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PREFACE.

A New Arithmetic! Why? Its brief history furnishes the answer.

During the four years last past, the author had under his especial charge this department in the Institute of which he was Principal, and a large portion of the time exclusively performed its duties.

As text books for his classes, many of the various popular works on the subject were, at different times, successively tried, and as many as were tried, were, for some cause, after a while, rejected;—some because the systems were *too new*, that is, too juvenile, as one reason; and in addition, with most of the others, as embracing too much, more than belongs properly to the subject; and, all dissatisfied, on one main account, which was, the almost total deficiency of expositions of principles, independent of numbers.

A consequence of the use of these books was found to be, in so far as reliance was placed on the books, confinement of the mind, rather than invigoration and development, and a reliance on formulas and rules, rather than on the great principles on which they were based; a stopping behind, rather than a looking within the veil.

The mind of the pupil was therefore confused; his views were all dependent and indistinct; his powers of analysis were never exercised vigorously; and his judgment was not made the arbiter of his work. Uncertainty as to results, unless the formulas and rules of his books were applied to square them, almost always characterized him. Therefore, when a practical question was presented, it was not uncommon for him to ask, To what rule does this belong? Instead of being made to see and know that but two principles can apply to numbers, and that they, variously applied, solve every question connected with them; and that the modes of application are always discoverable by an analysis to which the given question points; he felt that with each step of advancement a new principle was to be acquired, and so in learning a multiplicity of new things, out of two only which are as old as quantity itself and the first exercise of mind, became bewildered and disheartened.

The author then commenced writing and culling for his classes, bringing out the true and the necessary, and rejecting all else, with no idea of inflicting on the public the result of his efforts, further than they might be extended through his pupils.

But the labouriousness of writing and transcribing so much as was required daily, caused him to throw his matter in the first part into form, and put it in press, to supply the wants of his own school. When so far committed, in connection with the press, reasons multiplied rapidly for going on with what had been thus begun, and bringing out the whole subject.

To their variety, number, and force, the author yielded; and, as a consequence, now offers to the public a new Arithmetic, embracing old principles, as applied to their various purposes in connection with numbers, in the Troy Episcopal Institute, while he had that institution in charge.

The book might, perhaps, without impropriety, be termed *corsair;* for wherever, in the whole range of books upon the subject, any thing has been found suited to the purpose it has been unhesitatingly appropriated; and, as to *examples*, they have, in almost all cases, been taken from books before the public.

To the book of Lacroix, and to the American edition of it by Farrar, and to the truly excellent work of Hassler, the author acknowledges with gratitude his obligations. They are complete works on the subject; and the author of the present treatise, as their disciple, seeks not to be greater than or even as his masters.

While this general acknowledgement is made, it is hardly necessary to particularize and appropriate the parts in regard to which there is an especial obligation to one or the other.

In the present work there are, no doubt, many faults; some few have already come to the author's notice, and are subjoined as *errata;* and too much is said, perhaps, in illustration and explanation. Should another edition be called for, some of the defects of the present may be remedied. To this end the suggestions of teachers will be thankfully received.

An object especially aimed at in this work, has been to avoid the introduction of matter not pertinent to any practical use, and of that which more properly belongs to other departments of mathematical science. Hence, much that is embraced in most systems of Arithmetic is omitted. An obligation to encumber the work with Algebra, Geometry, Astronomy, Book-keeping, &c., &c., cannot be realized, while so many distinct treatises on those subjects are before the public, in accessible forms, and especially now that the subjects themselves are distinctly and separately taught in by far the larger portion of our schools.

Thus, it will be seen, that proportions are discarded entirely, and the principles of numbers analytically applied to the solution of questions usually referred to them, in a direct and intelligible form, which is substituted for various modes in all the remaining parts of the subject, except the roots.

Indeed the previous parts are regarded simply as preparatory to this, which is recommended to be substituted in all operations involving the true principles of numbers.

When this mode of operation has been once thoroughly acquired, its advantages will be so realized that it will hardly be exchanged, uniform, direct, and intelligible, as it will always be found, for the various modes in general use. It can require but a small share of experience to recommend one mode for many, one rule, short and always applicable and intelligible, for many, in regard to some of which at least, there is often perplexity in the choice and uncertainty in the results.

The principles of numbers can be only those of increase and diminution; the former invariably under the forms, addition and multiplication; the latter under those of subtraction and division. Why then should rules be so multiplied?

These principles are applied in this work, in Part I. to simple or abstract numbers; in Part II. to parts of simple numbers, or simple fractions; in Part III. to denominate numbers, or denominate fractions, as they are termed, after Mr. Hassler; and in Part IV. they are applied to all quantities indiscriminately; for which the other parts are, as already stated, only preparatory; Part V. embraces the roots.

Throughout the first three parts, the principles, and every application of them, are illustrated and explained; with these parts explanations cease, and the pupil is thrown on his own resources, to analyse and develope for himself, except in Part V.

The illustrations will in all cases guide the pupil in the special application of the

principles proposed, and to them alone it is expected the younger classes of pupils will have regard; omitting the explanations, which are abstract reasonings on stated applications of the principles, as presented in the cases and rules, to be analysed, and presented in substance, when more maturity and greater familiarity in operations under the rules, guided by the illustrations, are attained.

It is intended that the cases and rules should be thoroughly committed to memory; that the illustrations should be before the pupil, to show him how the application required is made; that the explanations should be mastered by analysis by older pupils; and the examples are for practice by all.

Before attempting this book, the pupil should become familiar with the Intellectual system of Colburn, a small work which cannot be too highly appreciated or recommended.

The constant reference to earlier modes of combination and to the reasons for them, in succeeding portions, by means of the sections and paragraphs, which are marked throughout the work, it is believed will be appreciated by the teacher, and be of great service to the pupil.

What the author has done for each subject, he need not state; he merely asks attention to each part, *beginning with notation*, and the intelligent teacher, and the practical man of business will discover wherein credit is due, and where not.

Hoping his efforts may contribute, in some measure, to facilitate the acquisition of the principles of this most general, most useful, and most important branch of science, the author commits their result to the public.

May 6, 1841.

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PART I.

ARITHMETIC.

§ 1. 1. MATHEMATICS is the science of Quantity.

2. QUANTITY is a term used to denote whatever admits of being increased, diminished, or measured.

3. NUMBERS are certain expressions for determinate quantity.

§ 2. 1. ARITHMETIC is a branch of mathematics which regards the properties and combinations of numbers.

2. Its principal operations are six; NOTATION, NUMERATION, ADDITION, MULTIPLICATION, SUBTRACTION, and DIVISION.

3. Four of these are essential and distinct; Notation, Numeration, Addition and Subtraction: the remaining two, Multiplication and Division, being embraced in two of the others, Multiplication in Addition, Division in Subtraction, are not essentially distinct operations, but are so treated for convenience.

I. NOTATION.

§ 3. 1. NOTATION is the writing of numbers.

2. It has two methods; one by characters or figures; the other by letters; known as the Arabic and Roman.

3. That by characters or figures, is the Arabic; that by letters, the Roman.*

* These methods are thus called because of their having been derived respectively from Arabia and Rome.

The Arabians were early devoted to scientific pursuits, which involved mathematics, and so were led to devise convenient and concise methods of expression. How early they invented and adopted the present characters is not accurately ascertained; but they were introduced into Europe in the ninth century.

The Romans devoted themselves rather to literature, and so were under no necessity to seek out a better mode of representing quantity than that which they have given us.

§ 4. 1. The *Arabic method* of Notation, for its conciseness and convenience, is that in most common use.

2. It employs ten figures, by various combinations of which, all determinate quantities may be expressed.

3. These figures are distinguished as *significant* and *insignificant*. The former, sometimes called *digits*,† are 1, 2, 3, 4, 5, 6, 7, 8, 9: the latter, usually called *cipher*, is 0.

Significant means with value; *insignificant* without value.

§ 5. 1. Each figure has its own *specific* or *simple*, and its *local* value.

2. The *simple* value of a figure is that which it has when it stands by itself.

3. The *local* value is a value that varies, according to the place in which the figure stands, when combined.

In a combination of figures, reckoning from right to left, the figure in the first place represents its *simple* value: those in the succeeding places have a *local* value.

§ 6. 1. Without combination, we cannot express more than *nine*.

2. To denote *ten*, a combination is made of the first *significant* figure (1) and *cipher* (0); the 1 being placed at the left of 0; thus, 10.

3. The increase is continued up to *nineteen*, by placing the significant figures at the right of the 1, in the room of the 0: thus, 11, 12, 13, 14, 15, 16, 17, 18, 19.

4. It is continued *one* farther, by increasing the figure at the left, and placing 0 at the right of it; thus, 20.

5. To continue the increase further, the significant figures are written in the same manner as before, (§ 6. 3); thus, 21, 22, 23, &c.: and so on up to 99.

6. After the nine digits have all in succession been used, at the left and right, (as in 99), *one* additional is expressed by placing 1 at the left of two ciphers; thus, 100; and a farther increase, by repeating with it the preceding combinations (§ 6. 5,) up to 199.

Then the figure at the left is increased, and, after that, those at the right; and so on up to 999.

7. For a farther increase, the 1 is removed yet further to the left, and the same combinations are repeated; and so on, indefinitely.

† These figures are doubtless called *digits*, from *digitus*, a finger, because counting used to be performed on the fingers.

§ 7. 1. When two or more figures are written by the side of each other, that on the right is called the *place of Units*, or *single ones;* that at the left, the *place of Tens;* the next, the *place of Hundreds;* the next, the *place of Thousands*. Then succeed *Tens of Thousands, Hundreds of Thousands, Millions, Tens of Millions, Hundreds of Millions;* and so on, through *Billions, Trillions, Quadrillions, Quintillions, Sextillions, Septillions, Octillions, Nonillions, Decillions, &c., &c.*

2. *Illustration.*

Hundreds of Quadrillions.	Tens of Quadrillions.	Quadrillions.	Hundreds of Trillions.	Tens of Trillions.	Trillions.	Hundreds of Billions.	Tens of Billions.	Billions.	Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

3. *Explanation.* Here 1, in the first place, at the right, signifies *one unit*, or *one;* 1, in the second place, *one ten*, or *ten;* 1, in the third place, *one hundred;* 1, in the fourth place, *one thousand;* 1, in the fifth place, *ten thousand;* 1, in the sixth place, *one hundred thousand;* and so on.

4. And such would be the value, or denomination of any figure written in any one of the places. 2 in the place of units, would be read simply 2; 2 in the *place of tens*, would be read *two tens*, or *twenty;* 2 in the *place of hundreds, two hundred;* and so on.

5. According to this system, the figures increase in value from right to left *tenfold.*

6. It is known as the *decimal system* of Notation.*

*This system of notation was probably obtained, through the Arabians, from India; and, in the ninth century, took the place of the Sexagesimal system previously used. The remains of this last system we yet have in the divisions of time, where *sixty* seconds make a minute; *sixty* minutes an hour, &c. There is no reason in the nature of numbers that their local value should vary according to this law. They might have been made to increase in 3, 4, 5, &c. fold, or in any other. The *tenfold* increase is assumed because it is most convenient. The circumstance of this increase taking place from right to left is owing to the fact,

7. Three places of figures, commencing at the right, form a period, to which a distinctive name is given. The periods are pointed off and named, UNITS, THOUSANDS, MILLIONS, BILLIONS, TRILLIONS, QUADRILLIONS, QUINTILLIONS, SEXTILLIONS, SEPTILLIONS, OCTILLIONS, NONILLIONS, DECILLIONS, &c., &c.†

8. *Illustration.*

10th period.	9th period.	8th period.	7th period.	6th period.	5th period.	4th period.	3d period.	2d period.	1st period.
of Octillions.	of Septillions.	of Sextillions.	of Quintillions.	of Quadrillions.	of Trillions.	of Billions.	of Millions.	of Thousands.	of Units.
369,	342,	900,	976,	368,	265,	371,	502,	634,	436.
Hundreds tens units	Hundreds tens units	Hundreds tens units	Hundreds tens units	Hundreds tens units	Hundreds tens units	Hundreds tens units	Hundreds tens units	Hundreds tens units	Hundreds tens units.

9. If no number of one or the other of these denominations is to be expressed, the place of it is supplied by a *cipher*, in order to preserve to each figure its proper place and value.

§ 8. 1. CASE I. *To write numbers by figures.*

RULE. Beginning at the left, write by periods, placing each period in its proper order; supplying by ciphers those places and periods that are omitted in the question.

2. *Illustration.*

Decillions.	Nonillions.	Octillions.	Septillions.	Sextillions.	Quintillions.	Quadrillions.	Trillions.	Billions.	Millions.	Thousands.	Units.
41.	080.	652.	941.	600.	807.	362.	546.	278.	009.	650.	208.

that the Arabians and Nations of the East always write from right to left instead of from left to right as we do.

† This is according to the French method. The English method gives six places to thousands, six to millions, &c.

3. *Examples.*

(1.) Three hundred and thirty-three.

(2.) Four hundred and forty-nine.

(3.) Five thousand, three hundred and twenty-five.

(4.) Six millions, four thousand and seventy.

(5.) Eighty-five millions, two thousand, four hundred and one.

(6.) One billion, one million, one thousand and one.

(7.) Ninety-five billions, eighty-one millions and seventy-five.

(8.) Two hundred and fifty quintillions, six quadrillions, two billions, three hundred and fifty thousand.

(9.) One hundred and twenty-three quintillions, one hundred and twenty-three billions, one hundred and twenty-three.

(10.) Two sextillions, three quintillions, four quadrillions, and four.

(11.) Eleven thousand, eleven hundred and eleven.

(12.) Fourteen thousand, fourteen hundred and fourteen.

(13.) One billion, two millions, four hundred and thirty-six.

(14.) Three hundred and nineteen thousand, and five hundred and fifty-five.

(15.) Seventeen millions, one thousand and forty-nine.

(16.) Four trillions, two billions, and seventy-four.

(17.) Eighteen thousand, three hundred and nine.

(18.) One quadrillion, four thousand and two hundred.

(19.) One hundred and six trillions, four thousand and two.

(20.) Fifty-nine trillions, fifty-nine billions, fifty-nine millions, fifty-nine thousand, and fifty-nine.

4. The formality of separating the periods, by a point, need not be observed, when the pupil has become familiar with the principles of Notation and acquired a facility in applying them.

§ 9. 1. It will be well for the teacher to illustrate the remark in the note on page 11, that there is no reason in the nature of numbers why the *tenfold* system of increase in the value of figures, from right to left, should be used rather than any other.

2. It may at once be seen, that other systems might be

formed upon the same principles, and, of course, with the same properties, as to the expression of the greater quantities by the successive rank or place of the figures; and with any other greater or smaller number of significant figures; only that cipher (0) and unit (1) must make part of every such system.

3. If no other figures were used but 1 and 0, the value in each would always be successively *double* of that in the preceding place to the right; and so the whole of the calculation would become a mechanical change of places. The following numbers, 111101, transcribed into our usual decimal system, would be 32, 16, 8, 4, and 1, or 61.

4. If three figures were used, (1, 2, 0,) the increase from right to left would be *threefold;* the numbers 11101, transcribed into our decimal system, would be 81, 27, 9, and 1, or 118. 20347 would be 208; and so on.

5. So a fourfold, fivefold, sixfold, or any other system might be adopted.

6. The decimal system is, however, most convenient, as is apparent, and is, therefore, in universal use.

§ 10. 1. The *Roman method* of Notation is by letters.

2. It employs seven letters, I, V, X, L, C, D, M. I, represents *one;* V, *five;* X, *ten;* L, *fifty;* C, *one hundred;* D, *five hundred;* M, *one thousand.*

3. No other number can be expressed by single letters alone.

4. When any other number is to be expressed, a combination is made of the seven letters above, or their value is varied by a sign.

5. A letter placed at the *right* of one representing *an equal* or *greater* value, denotes that the letter at the left *is increased* by so much; a letter placed *at the left* of one of a *greater value*, denotes that so much *is taken from that at the right.*

§ 11. 1. CASE II. *To express numbers by letters.*

RULE. Write first the letter which expresses the number nearest that which is to be written. If that be too small, write other letters at the right to increase it, (§ 10. 5); or, if too large, write other letters at the left to reduce it to the given number, (§ 10. 5).

2. *Illustration.*

I.	One.	LX.	Sixty.
II.	Two.	LXX.	Seventy.
III.	Three.	LXXX.	Eighty.
IV.	Four.	XC.	Ninety.
V.	Five.	C.	One hundred.
VI.	Six.	CC.	Two hundred.
VII.	Seven.	CCC.	Three hundred.
VIII.	Eight.	CCCC.	Four hundred.
IX.	Nine.	D.	Five hundred.
X.	Ten.	DC.	Six hundred.
XX.	Twenty.	DCC.	Seven hundred.
XXX.	Thirty.	DCCC.	Eight hundred.
XL.	Forty.	DCCCC.	Nine hundred.
L.	Fifty.	M.	One thousand.

3. IↃ is sometimes used instead of D, to represent five hundred, and for every additional Ↄ annexed at the right, the number is increased *ten times;* CIↃ to represent one thousand, and for every C and Ↄ put at each end, the number is increased *ten times.* A line over any number increases its value *one thousand times.* Thus, IↃↃ, or $\overline{\text{V}}$, five thousand; $\overline{\text{M}}$, one million.

4. By this method of combining letters they may be made to denote all determinate quantities.

5. *Examples.*

(1.) Nine hundred and forty-three.

(2.) One thousand, eight hundred and fifty-six.

(3.) Two millions, four hundred and twenty-five thousand, four hundred and eleven.

(4.) One billion, two hundred millions, forty thousand, three hundred and fifteen.

(5.) Twenty-five trillions, one hundred and four billions, three hundred and seventy-five millions, and fifty-four.

(6.) Five hundred and thirty-seven millions, two hundred and nineteen thousand, one hundred and three.

(7.) Seven hundred and seventy-seven millions, seven hundred and seven.

(8.) Fourteen thousand, nine hundred and seventy-eight.

(9.) Thirty-two millions, twenty-three thousand, two hundred and fifty-six.

II. NUMERATION.

§ 12. 1. NUMERATION is the reading of numbers.

2. It is a mere declarative act, consisting, when the numbers are expressed by figures, in calling off each figure by the appropriate name of the place which it occupies; when by letters, in calling off the entire number.

§ 13. 1. CASE I. *To read numbers expressed by figures.*

RULE. Beginning at the left, call each significant figure in order, by the appropriate name of its place and period, except the last, to which the name is not added.

2. *Illustration.*

Decillions.	Nonillions.	Octillions.	Septillions.	Sextillions.	Quintillions.	Quadrillions.	Trillions.	Billions.	Millions.	Thousands.	Units.
41.	080.	652.	941.	600.	807.	362.	546.	278.	009.	650.	208;

41.080.652.941.600.807.362.546.278.009.650.208; which is read, forty-one decillions, eighty nonillions, six hundred and fifty-two octillions, nine hundred and forty-one septillions, six hundred sextillions, eight hundred and seven quintillions, three hundred and sixty-two quadrillions, five hundred and forty-six trillions, two hundred and seventy-eight billions, nine millions, six hundred and fifty thousands, two hundred and eight.

3. *Examples.* To be written in words, (equivalent to reading:)

(1.) 49.
(2.) 356.
(3.) 9758.
(4.) 7004.
(5.) 268012.
(6.) 863700523.
(7.) 701001204.
(8.) 345068900457.
(9.) 12344567890.
(10.) 387504218756324.
(11.) 11010001000001.
(12.) 999999899979996993991.

6. The numeration of numbers, expressed by letters, is sufficiently obvious, from the directions given for their notation (§ 10. 5). But to carry out the uniformity of our plan, we subjoin a distinct Case and Rule.

§ 14. 1. CASE II. *To read numbers expressed by letters.*

RULE. Begin at the left, and call each letter by name, in order; by the rule for notation by letters, ascertain the number expressed, and declare it.

2. *Illustration.* MDCCCXLI.
Here we begin at the left, and say, M, D, three Cs, XL, I: one thousand eight hundred and forty-one.

3. *Examples.* To be written in words (equivalent to reading):

(1.) XL.
(2.) LVI.
(3.) LXXIX.
(4.) CXCIX.
(5.) DCCCLXVII.
(6.) $\overline{\text{M}}\overline{\text{D}}$CIↃ.
(7.) $\overline{\text{M}}$MDC.
(8.) $\overline{\text{IↃ}}$.
(9.) IↃↃDLV.
(10.) $\overline{\text{V}}\overline{\text{C}}$IↃLXXXIV.
(11.) MDCLXVI.
(12.) MMCCCCXCXIX.

4. In Latin authors, the Roman method of notation is alone met with.

5. By us, at present, its chief use is to mark divisions and subdivisions in books; and sometimes the number of the year.

6. To express quantity, for Arithmetical calculations, it is not at all employed.

No further reference to it, therefore, will be made in this work; the Arabic method alone will be understood and employed.

III. ADDITION.

§ 15. 1. Addition is the bringing together of two or more numbers into one. Thus, 4 and 3, may be brought together, into the one number 7.

2. The result of an addition, is called *sum*, *total*, or *amount*.

3. Addition is indicated by a sign, consisting of a horizontal line (—) crossed by a perpendicular (|), (+), called *plus*, which signifies more. 4+3, denotes that 3 is to be added to 4.

4. Equality is denoted by two horizontal lines (=), which are read *equals*, or *is equal to*. 4+3=7, denotes that 3 added to 4 equals 7.

§ 16. 1. Case. *To add numbers.*

Rule. I. Write the numbers under each other, in the order of, units under units, tens under tens, and so on; and draw a horizontal line underneath.

II. Begin at the foot of the right hand column, and bring together all the units it contains.

If the sum be less than 10, write it under the units; if it equal or exceed 10, write, under the units, the figure at the right, and keep in memory, that at the left (which is a *ten* or *tens*, § 7. 1, 2, 3), to be added to the *tens* in the next column.

III. Bring together the tens in the next column, and, add to the sum, the *ten* or *tens* kept in memory, taken out of the preceding column.

If the sum be less than 10, write it under the *tens;* if it equal or exceed 10, write the figure at the right under the *tens*, and keep in memory, that at the left, to be added to the *hundreds* (§ 7. 1, 2, 3) in the next column.

IV. Proceed in the same manner with each succeeding column to the last, the whole sum of which write down.

2. *Illustration.* 67421+389+641827+30+4+7259.

(1.) We write the numbers under each other, units under units, tens under tens, and so on; and draw a horizontal line under them.

67421
389
641827
30
4
7259
716930

(2.) We then begin at the foot of the column of units, and add, saying, 9+4+0+7+9+1=30; we write 0 under the units, and keep the 3 (which, being in the second place (§ 7. 1), is *tens*) in memory, to add to the *tens* in the next column—that of tens.

(3.) Then for the next column, that of the tens, we say, 3 (for the 3 kept in mind) +5+3+2+8+2=23; we write 3 under the tens, and keep the 2 (which, being in the third place, is *hundreds*, § 7. 1) in mind, to be added to the next column—that of hundreds.

(4.) We proceed in this manner to the last figure on the left hand, when the true sum, 716930, is obtained.

3. *Explanation.* The Rule is founded on the principle of Notation, that figures increase in value from right to left *tenfold* (§ 7. 5). Taking on in memory, therefore, the figure at the left, in each case when the sum equals or exceeds 10, is but bringing together those of the same value. Its correctness is evident from the nature of Notation (§ 7. 5). It is called *carrying*.

4. The accuracy of an operation, in Addition, may be tested, by adding downwards. If the sums agree, there is probably no error; as, we should not be likely to commit the same error, adding two different ways.

5. *Proof.** Addition is proved by reversing the operation; that is, by taking away, from the *sum*, the numbers put together to make it. This may be done by successively subtracting each of the numbers added; the first from the sum, the next from that remainder, and the next from that, and so on to the last, which, if the operation be correct, will leave no remainder.

* This proof should be omitted till the pupil has become familiar with Subtraction.

Or, shorter; the numbers may be added from the left, and the sum of each column be subtracted from the sum first obtained; the remainder, after each subtraction, being supposed to stand at the left of each succeeding figure in the *sum*, and to be added to it.

If, with this addition, the sum of each column can be subtracted from that of each column first obtained, and so on to the last, and then there be no remainder, the operation is correct.

6. *Illustration.*

```
 7346
 6235
 5124
 4013
 3602
 ----
26320
 ----
 1120
```

(1.) Here we add, at first, the figures contained in the column at the left, which consists of thousands, and subtract the sum 25, from the number 26, and set down the difference 1. This 1 is produced from what was carried on from the next lower column, that of hundreds, in the first addition.

(2.) We then suppose the 1 to be added to the next figure in the sum, to that in the place of hundreds, which is 3; and which, by the addition, becomes 13; or, as the 1, in reality, is 1000, and the 3, 300, the amount is 1300.

(3.) We then add the column of hundreds, the sum of which is 12, or 1200, subtract it from the 13, or 1300, and we have a remainder 1. This again is produced from what was carried on from the next lower column, that of tens, in the first addition.

(4.) We then suppose it added to the next figure in the *sum*, that in the place of tens, which is 2, and which, by the addition, becomes 12; or, as the 1, in reality, is 100, and the 2, tens, or twenty, the amount is 120.

(5.) We then add the column of tens, the sum of which is 10, or 100, subtract it from the 12, or 120, and we have a remainder 2. This is produced from what was carried from the units, in the first addition.

(6.) We then suppose this 2, which is, in reality, 2 tens, to be added to the next figure in the *sum*, that in the unit's place, which is 0, and which, by the addition, becomes 20.

(7.) Then we add again the units, the sum of which is 20, which subtracted from the sum last obtained, leaves no remainder.

(8.) The operation, therefore, was correctly performed.

7. *Explanation.* Addition and Subtraction are directly opposite one to the other. Each, therefore, must prove the other. Bringing together numbers separated by subtraction, must, if the operation be correctly performed, give the number from which the subtraction was made, and so prove the operation of subtraction (§ 34. 1). Separating numbers brought together by addition, from the sum in which they are united, must, if those numbers are exactly separated from that sum, prove the operation of addition.

The remainder, in each case taken to the right, is produced from what was carried on to the left, in the first addition. And, in this reverse operation, it is obvious, it should be taken back where it was first obtained.

8. *Examples.*

(1.) Add 94372+82354+79601+18294+52987+651.
Ans. 328259.

(2.) Add 5687450369+6798560673+8709293461+1324658709+4576349258. *Ans.* 27096312470.

(3.) Add 235809+420+72910+509267+8392.
Ans. 826798.

(4.) Add 83081+16032+52071+73065+71054+640.
Ans. 295943.

(5.) Add 91704+27541+50179+73276+58403.
Ans. 301103.

(6.) A man gave to one of his sons 435 dollars; to another 629; and to another 747; how many did he give them all? *Ans.* 1811.

(7.) A gentleman purchased a farm for 5941 dollars, paid 375 dollars for fencing it, and 290 dollars for having a barn built upon it. For how many dollars must he sell it, to gain 100 dollars? *Ans.* 6706.

(8.) A merchant, on settling his accounts, finds he owes

A. 499 dollars; B. 537; C. 626; D. 792; and E. 870. How much does he owe them all? *Ans.* 3324.

(9.) What is the sum of four, forty, four hundred, four thousand, forty thousand, four hundred thousand, four millions, forty millions, and four hundred millions?

Ans. 444444444.

(10.) What is the sum of three hundred and forty-seven, six thousand three hundred, five hundred and twenty-five, twelve thousand four hundred and ninety-eight, two millions nine hundred and eighteen, and one hundred and ninety-nine thousand seven hundred and forty-three?

Ans. 2220331.

(11.) What is the sum of nineteen, ninety, seven hundred and twenty-nine, five thousand four hundred and fifty-six, eighteen thousand and thirty-five, and three hundred and twenty thousand nine hundred and eighty-one?

Ans. 345310.

(12.) A gentleman, being asked how old he was, said, he was married when he was 29 years of age; that he lived with his wife 8 years before the birth of their son, who was now 27 years of age. What was his age?

Ans. 64 years.

(13.) Bought of my neighbor 4 loads of hay; the first, weighed 1600 pounds; the second, 2100 pounds; the third, 1999 pounds; the fourth, 1709 pounds. What was the whole weight? *Ans.* 7408 pounds.

(14.) For a certain undertaking in a village, seven men agree to give, each as much money as he has cash in pocket; John has 47 dollars; Peter, 121; James, 50; Richard, 79; Francis, 107; Frederick, 192; and William, 305. How much stock do they bring together?

Ans. 901 dollars.

(15.) An orchard contained 325 apple trees, 300 pear trees, 412 peach trees, 46 plum trees, 5 cherry trees, and 6000 mulberry trees. How many trees did the orchard contain? *Ans.* 7088 trees.

(16.) A merchant had a store-house in which he had, at one time, 6000 bushels of oats, 5765 bushels of wheat, 1375 bushels of rye, 8750 bushels of corn, and had room enough left for 2000 bushels more of corn. How many bushels would the store-house hold?

Ans. 23890 bushels.

IV. MULTIPLICATION.

§ 17. 1. MULTIPLICATION is the bringing together as many times one number as is denoted by another.

2. It is not an essentially distinct operation; but is simply a convenient and concise method of performing Addition. Thus, if the sum of 4 times 5 were required, the mode by addition would be to write the 5 four times,

$$\begin{array}{r} 5 \\ 5 \\ 5 \\ 5 \\ \hline 20: \end{array}$$

by multiplication, the 5 is written but once, with the 4, denoting the repetition, under it,

$$\begin{array}{r} 5 \\ 4 \\ \hline 20. \end{array}$$

3. The number repeated is called the *multiplicand;* that denoting the repetition, the *multiplier;* both together, considered as concurring to produce the result, *factors;* the result, the *product.*

4. The exact product of two or more numbers is, sometimes, called a *composite number;* and, sometimes, a *multiple.*

5. Multiplication is indicated in two ways; by a sign, consisting of a line drawn oblique to the right (\\) crossed by one drawn oblique to the left (/), (×), called the sign of multiplication; and by a period (.). 4×3, denotes that 4 is to be multiplied by 3; so 4.3.

6. These and the other signs employed in Arithmetic, are of great practical use, in denoting and abridging operations. Their employment must be left to the good sense and dexterity of the pupil, like that of many other modes of abridging Arithmetical calculations, the pointing out of which would involve unnecessary and tedious details.

7. Before the operation of Multiplication can be performed, the pupil must thoroughly commit to memory the

following, from its supposed inventor, called *Pythagorean*, or, more commonly,

MULTIPLICATION TABLE.

2 times 1 are 2	5 times 1 are 5	8 times 1 are 8
2 times 2 are 4	5 times 2 are 10	8 times 2 are 16
2 times 3 are 6	5 times 3 are 15	8 times 3 are 24
2 times 4 are 8	5 times 4 are 20	8 times 4 are 32
2 times 5 are 10	5 times 5 are 25	8 times 5 are 40
2 times 6 are 12	5 times 6 are 30	8 times 6 are 48
2 times 7 are 14	5 times 7 are 35	8 times 7 are 56
2 times 8 are 16	5 times 8 are 40	8 times 8 are 64
2 times 9 are 18	5 times 9 are 45	8 times 9 are 72
2 times 10 are 20	5 times 10 are 50	8 times 10 are 80
2 times 11 are 22	5 times 11 are 55	8 times 11 are 88
2 times 12 are 24	5 times 12 are 60	8 times 12 are 96

3 times 1 are 3	6 times 1 are 6	9 times 1 are 9
3 times 2 are 6	6 times 2 are 12	9 times 2 are 18
3 times 3 are 9	6 times 3 are 18	9 times 3 are 27
3 times 4 are 12	6 times 4 are 24	9 times 4 are 36
3 times 5 are 15	6 times 5 are 30	9 times 5 are 45
3 times 6 are 18	6 times 6 are 36	9 times 6 are 54
3 times 7 are 21	6 times 7 are 42	9 times 7 are 63
3 times 8 are 24	6 times 8 are 48	9 times 8 are 72
3 times 9 are 27	6 times 9 are 54	9 times 9 are 81
3 times 10 are 30	6 times 10 are 60	9 times 10 are 90
3 times 11 are 33	6 times 11 are 66	9 times 11 are 99
3 times 12 are 36	6 times 12 are 72	9 times 12 are 108

4 times 1 are 4	7 times 1 are 7	10 times 1 are 10
4 times 2 are 8	7 times 2 are 14	10 times 2 are 20
4 times 3 are 12	7 times 3 are 21	10 times 3 are 30
4 times 4 are 16	7 times 4 are 28	10 times 4 are 40
4 times 5 are 20	7 times 5 are 35	10 times 5 are 50
4 times 6 are 24	7 times 6 are 42	10 times 6 are 60
4 times 7 are 28	7 times 7 are 49	10 times 7 are 70
4 times 8 are 32	7 times 8 are 56	10 times 8 are 80
4 times 9 are 36	7 times 9 are 63	10 times 9 are 90
4 times 10 are 40	7 times 10 are 70	10 times 10 are 100
4 times 11 are 44	7 times 11 are 77	10 times 11 are 110
4 times 12 are 48	7 times 12 are 84	10 times 12 are 120

11 times 1 are 11	12 times 1 are 12
11 times 2 are 22	12 times 2 are 24
11 times 3 are 33	12 times 3 are 36
11 times 4 are 44	12 times 4 are 48
11 times 5 are 55	12 times 5 are 60
11 times 6 are 66	12 times 6 are 72
11 times 7 are 77	12 times 7 are 84
11 times 8 are 88	12 times 8 are 96
11 times 9 are 99	12 times 9 are 108
11 times 10 are 110	12 times 10 are 120
11 times 11 are 121	12 times 11 are 132
11 times 12 are 132	12 times 12 are 144

§ 18. 1. CASE 1. *To multiply by a single figure.*

RULE I. I. Write the multiplier under the units of the multiplicand, and draw a horizontal line underneath.

II. Beginning with the units, multiply each figure in the multiplicand, successively to the last, writing under each its product, if it be less than 10; if it equal or exceed 10, writing down the right hand figure, keeping that at the left in mind to be added to the next higher product (§ 7. 1).

III. The entire product of the last figure write down.

2. *Illustration.* 4789×7.

$$\begin{array}{r} 4789 \\ 7 \\ \hline 33523 \end{array}$$

(1.) We write the multiplier under the units of the multiplicand, and draw a horizontal line below.

(2.) We then begin at the right, and say, 7 times 9 are 63; we write 3 and keep the 6 (which is *tens*, § 7. 1.) in memory, to be added to the next higher product—that of the tens.

(3.) We then say, for the next figure, the tens, 7 times 8 are 56; to which we add the 6 kept in mind from the last product, which gives 62; we write the 2, and keep the 6 (which is *hundreds*) in mind, to be added to the next higher product—that of the hundreds; and so on to the last figure; the whole product of which we write down, and the true product, 33523, is obtained.

3. *Explanation.* Multiplying each figure in the multiplicand by the multiplier, is precisely equivalent to an addition of so many times the multiplicand as is denoted by the multiplier. Thus, 4789 written seven times and added, gives a *sum* just equal to the *product* in the illustration.

$$\begin{array}{r} 4789 \\ 4789 \\ 4789 \\ 4789 \\ 4789 \\ 4789 \\ 4789 \\ \hline 33523 \end{array}$$

4. Multiplication, therefore, being but the addition of the same quantity a certain number of times, it is obvious why the left hand figure in each product should be added to the next higher product (§ 7. 5).

The reason for this is the same as that for carrying in Addition (§ 16. 3).

5. Or, RULE II. Indicate the operation by the sign; draw a horizontal line underneath; multiply as before, and write the product below the line.

7. *Illustration.*

$$\begin{array}{r} 4789\times3 \\ \hline 14367 \end{array}$$

8. Or, RULE III. Indicate the operation by the sign; write after the multiplier the sign of equality; multiply as before, and write the product at the right.

9. *Illustration.* $4789\times3=14367$.

10. *Examples.*

(1.) Multiply 3579 by 4. *Ans.* 14316.
(2.) Multiply 53182 by 6. *Ans.* 319092.
(3.) Multiply 11472 by 5. *Ans.* 57360.
(4.) Multiply 293815 by 3. *Ans.* 881445.
(5.) Multiply 862537 by 7. *Ans.* 6037759.
(6.) Multiply 4136928 by 8. *Ans.* 33095424.

(7.) Multiply 9425738 by 9. *Ans.* 84831642.
(8.) Multiply 37146927 by 5. *Ans.* 185734635.
(9.) Multiply 741137913 by 8. *Ans.* 5929103304.
(10.) Multiply 397564257 by 6. *Ans.* 2385385542.
(11.) If a man earn 3 dollars a day, how many will he earn in 52 days? *Ans.* 156 dollars.
(12.) If a man thrash 8 bushels of rye a day, how many will he thrash in 46 days? *Ans.* 368 bushels.
(13.) What will 765 pounds of cheese cost, at 9 cents per pound? *Ans.* 6885 cents.
(14.) What will 894 sheep cost, at 4 dollars a head?
Ans. 3576 dollars.
(15.) What will 1582 feet of timber cost, at 7 cents a foot? *Ans.* 11074 cents.
(16.) A father distributed his property equally among 6 sons. Each son's part was 56347 dollars. What was the value of the father's property? *Ans.* 338082 dollars.
(17.) Says John to Dick, you have 8 times 27 marbles, and I have 5 times as many as you. How many had John? *Ans.* 1080 marbles.
(18.) A merchant bought 33 bales of cloth, each bale containing 8 pieces, and each piece 9 yards. How many yards were there in all? *Ans.* 2376 yards.
(19.) How many days will it take 1 man to do a piece of work, that 9 men will do in 72 days?
Ans. 648 days.

§ 19. 1. Case II. *To multiply by* 10, 100, 1000; *or by* 1 *with any number of ciphers annexed.*

Rule. Annex the ciphers of the multiplier to the multiplicand.

2. *Illustration.*

(1.) 4869×10=48690. (2.) 4869×100=486900.

3. *Explanation.* Each cipher annexed to the multiplicand, removes it one place farther to the left; putting units in the place of tens, tens in that of hundreds, and so on; and so increases it tenfold (§ 7. 5).

4. *Examples.*

(1.) Multiply 347 by 10. *Ans.* 3470.
(2.) Multiply 561 by 100. *Ans.* 56100.
(3.) Multiply 983 by 1000. *Ans.* 983000.
(4.) What will 76 barrels of flour cost, at 10 dollars a barrel? *Ans.* 760 dollars.
(5.) If 100 men receive 291 dollars each, how many will they all receive? *Ans.* 29100 dollars.
(6.) What will 100 pieces of broadcloth cost, estimating each piece at 273 dollars? *Ans.* 27300 dollars.

§ 20. 1. CASE III. *To multiply by two or more significant figures.*

RULE. I. Write the multiplier under the multiplicand, in the order of, units under units, tens under tens, and so on, and draw a horizontal line underneath.

II. Beginning with the units, multiply each figure in the multiplicand by each figure in the multiplier successively, and write the first figure of each product directly under that figure of the multiplier by which it is produced, and carry as in Case I. Rule I.

III. Add together the products of the several figures of the multiplier, and their sum will be the product required.

2. *Illustration.*

$$7458 \times 423 \qquad \begin{array}{r} 7458 \\ 423 \\ \hline 22374 \\ 14916 \\ 29832 \\ \hline 3154734 \end{array}$$

(1.) Here we write the multiplier under the multiplicand, in the order of units under units, and so on.

(2.) We then begin with the units, in the multiplier, and multiply each figure in the multiplicand by it, as in Case I.

(3.) We then take the 2, in the tens place in the multiplier, and multiply each figure in the multiplicand by it, in

the same way, only that the multiplier being *tens*, the product is written to the left one place, to increase it tenfold (§ 7. 5), and so the first figure is written in the tens place, directly under that figure in the multiplier which produced it.

(4.) We then take the 4, in the hundreds place, multiply by it in the same way, and write the first figure of the product in the place of hundreds, directly under that figure in the multiplier which produced it.

(5.) The sum of the three products, thus obtained, is the required product, 3154734.

3. *Explanation.* Each significant figure in the multiplier, after the first to the right, has a local value (§ 5. 3) which is determined by its place. The product of each must, therefore, be so many fold greater than that by simple units as will correspond with the figure of the multiplier by which it is produced. It is so increased by removing it so many places to the left as that occupied by the multiplying figure (§ 7. 5); or, in other words, by placing the first figure of each product directly under that figure of the multiplier by which it is produced (§ 18. 3).

4. It is obviously immaterial which figure of the multiplier is multiplied by first, if the rule for placing each product be carefully observed (§ 20. 1. Rule. II.).

5. The right hand figure is usually taken first for convenience.

6. *Examples.*

(1.) Multiply 756 by 78. *Ans.* 58968.
(2.) Multiply 517 by 89. *Ans.* 46013.
(3.) Multiply 436 by 133. *Ans.* 57988.
(4.) Multiply 76432 by 2345. *Ans.* 179233040.
(5.) Multiply 968753 by 7953. *Ans.* 7704492609.

(6.) A merchant sold 346 tons of iron, at 142 dollars per ton. What was the price of the whole?
Ans. 49132 dollars.

(7.) There are 24 hours in a day, and 365 days in a year. If a ship sail 7 miles in an hour, how many miles will she sail in a year? *Ans.* 61320 miles.

(8.) What is the value of the hay that is produced on 19 farms; allowing each farm to produce 75 tons, and allowing the hay to be worth 12 dollars a ton?
Ans. 17100 dollars.

§ 21. 1. CASE IV. *To multiply, when a cipher or ciphers intervene between the significant figures, either of the multiplicand or multiplier.*

RULE. I. Write down the numbers as in Case III., and proceed, as in that case, with the multiplication of the significant figures.

II. If cipher occurs in the multiplicand, write, in the place for its product, cipher only, or that number which may be carried from the preceding multiplication; if it occurs in the multiplier, pass it over; taking care that the first figure of the next product stand directly under its multiplier.

2. *Illustrations*,

(1st.) 4807×23	4807	(2d.) 3876×4007	3876
	23		4007
	14421		27132
	9614		15504
	110561		15531132

(1.) In the first of these illustrations, cipher occurs in the multiplicand; and, in multiplying, its place receives, when we multiply by the 3, 2 which is carried on from the multiplication of the units; when we multiply by the 2, its place receives 1, which is carried on from the preceding multiplication.

(2.) In the second illustration, we have two ciphers in the multiplier, both of which are entirely passed over; their only influence being to remove the next product, that produced by the 4, two places to the left.

3. *Explanation.* The product of cipher by a significant figure is cipher only; so the product of a significant figure by cipher is the same; therefore, when cipher occurs in either or both of the factors, it is passed over (§ 4. 3).

For the multiplication of the significant figures, the explanation is the same as that to Case III.

4. *Examples.*

(1.) Multiply 4976 by 108. *Ans.* 537408.

(2.) Multiply 3742 by 104. *Ans.* 389168.
(3.) Multiply 23243 by 2002. *Ans.* 46532486.
(4.) Multiply 34567 by 4001. *Ans.* 138302567.
(5.) Multiply 90042 by 9009. *Ans.* 811188378.
(6.) Multiply 701231 by 1231. *Ans.* 863215361.

(7.) If a ship sail at the rate of 109 miles a day, how many miles will she sail in 72 days? *Ans.* 7848 miles.

(8.) Suppose 42 men were concerned in the payment of a debt, and each man paid 1034 dollars. How much was the debt? *Ans.* 43428 dollars.

(9.) Suppose an orchard to consist of 109 rows, 126 trees in a row, and 1007 apples on a tree; how many trees, and how many apples are there?
Ans. 13734 trees, 13830138 apples.

§ 22. 1. CASE V. *To multiply, when there are ciphers at the right of either or both of the factors.*

RULE. Write the significant figures of the multiplier under those of the multiplicand, multiply by them, and annex the ciphers of both factors to the product.

2. *Illustrations.*

(1.) 371×230

$$\begin{array}{r} 371 \\ 230 \\ \hline 1113 \\ 742 \\ \hline 85330 \end{array}$$

(2.) 5700×490

$$\begin{array}{r} 5700 \\ 490 \\ \hline 513 \\ 228 \\ \hline 2793000 \end{array}$$

3. *Explanation.* When the ciphers at the right of the factors are disregarded, each factor is diminished tenfold for every cipher omitted, by having its significant figures brought so many places to the right. The product must then be increased so many times tenfold as there were ciphers omitted. This is done, by annexing the ciphers to the product (§ 19. 3).

4. *Examples.*

(1.) Multiply 462 by 250. *Ans.* 115500.
(2.) Multiply 537 by 340. *Ans.* 182580.

(3.) Multiply 3520 by 460. *Ans.* 1619200.
(4.) Multiply 683500 by 59300. *Ans.* 40531550000.
(5.) Multiply 789300 by 6840. *Ans.* 5398812000.
(6.) Multiply 918270 by 72000. *Ans.* 66115340000.
(7.) There are 320 rods in a mile; how many rods are there in 436 miles? *Ans.* 139520 rods.
(8.) What is the value of a farm consisting of 280 acres of land, at 140 dollars an acre? *Ans.* 39200 dollars.
(9.) A man had a farm, on which he raised 360 bushels of wheat; and he had another, on which he raised 6 times as much; what quantity did he raise on both? *Ans.* 2520 bushels.
(10.) If a man travel 40 miles in a day, how many miles will he travel in 236 days? *Ans.* 9440 miles.

§ 23. 1. CASE VI. *To multiply by* 9, 99, 999, *or by any number of nines.**

RULE. Annex to the multiplicand as many ciphers as there are *nines* in the multiplier; and, from the product, subtract once the multiplicand.

2. *Illustrations.*

(1.) 473×9	4730	(2.) 398×99	39800
	473		398
	4257		39402

3. *Explanation.* Annexing a cipher to the multiplicand, multiplies it by 10 (§ 19. 3); two ciphers annexed, multiplies it by 100; and so on; and so the multiplicand is taken *once* more than is required when the multiplier is 9, or any number of nines. It is, therefore, subtracted once.

4. *Examples.*

(1.) Multiply 48 by 9. *Ans.* 432.
(2.) Multiply 362 by 9. *Ans.* 3258.
(3.) Multiply 563 by 99. *Ans.* 55737.
(4.) Multiply 6473 by 99. *Ans.* 640827.

* This case should be passed over till the pupil has learned Subtraction.

(5.) Multiply 5821 by 999. *Ans.* 5815179.
(6.) Multiply 4629 by 999. *Ans.* 4624371.
(7.) Multiply 345 by 9999. *Ans.* 3449655.
(8.) A certain cornfield contains 228 rows, which are 99 hills long. How many hills are there?
Ans. 22572 hills.
(9.) If a man's income be 1 dollar a day, what will be the amount of his income in 99 years, allowing 365 days to each year? *Ans.* 36135 dollars.
(10.) In dividing a certain sum of money among 89 men, each man received 999 dollars. What was the sum divided? *Ans.* 88911 dollars.

§ 24. 1. CASE VII. *To multiply by a composite number* (§ 17. 4).

RULE. Indicate the operation by the sign; multiply first by one of the factors, and that product by the other; the last product will be the product required.

2. *Illustration.*

$$5937\times48. \quad 48=8\times6 \qquad \begin{array}{r}5937\\8\\\hline 47496\\6\\\hline 284976\end{array} \quad \text{or,} \quad \begin{array}{r}5937\times8\\\hline 47496\times6\\\hline 284976\end{array}$$

3. *Explanation.* In the illustration, the product of the multiplicand, by 48, is equal to 6 times its product by 8, because 48 is 6 times as many as 8. And the reason, it is evident, will apply to all operations under the rule; it will hold universally, that the multiplication of the multiplicand by one factor of the multiplier and that product by the other; and, if there be more factors, that product by the next, and so on; will give a final product precisely equivalent to that obtained by the whole multiplier.

4. *Examples.*

(1.) Multiply 183 by 49. *Ans.* 8967.
(2.) Multiply 262 by 63. *Ans.* 16506.

(3.) Multiply 893 by 84. *Ans.* 75012.
(4.) Multiply 2784 by 96. *Ans.* 267264.
(5.) Multiply 510361 by 36. *Ans.* 18372996.

(6.) Bought 75 tons of hay, at 15 dollars a ton. What did the whole cost? *Ans.* 1125 dollars.

(7.) What cost 172 acres of land, at 36 dollars an acre? *Ans.* 6192 dollars.

(8.) What will 876 pounds of coffee cost, at 28 cents per pound? *Ans.* 24528 cents.

§ 25. 1. Case VIII. *To multiply, when the multiplier has in it a multiple, embracing one or more of its remaining figures.*

Rule. Indicate the operation by the sign; multiply first by that figure of the multiplier which is embraced in the multiple, and that product by the other factor in the multiple; writing the first figure of each product directly under the right hand or lowest figure of the number whose product it is to denote.

2. *Illustrations.*

(1st.) 897×618.

```
   897
   618
 -----
  5382 ×3
 16146
 -----
554346
```

(2d.) 7351×637.

```
    7351
     637
 -------
   51457×9
 463113
 -------
 4682587
```

(3d.) 64793×85672.

```
     64793
     85672
 ----------
    518344  ×7×9
   3628408
  4665096
 ----------
 5550945896
```

(1.) In the first illustration, 18 is the multiple of the remaining figure 6; 6×3=18. We then multiply by the 6, the number of the multiplier embraced in the multiple; and that product by 3, the other factor in the multiple,

writing the product two places to the right, its first figure under the 8, to denote the product of 18 (§ 24. 3).

(2.) In the second illustration, the multiple is 63; the factor expressed is 7. We multiply first by the 7, and its product by 9, the other factor in the multiple; 7×9=63 (§ 24. 3).

(3.) In the third illustration, we have two multiples, 56 and 72; 56=8×7; 72=9×8. We multiply first by 8, the factor in 56 expressed, and that product by 7, the other factor in 56; then the 8, being also a factor in 72, we multiply its product by 9, the other factor in 72. For the propriety of multiplying by the factors of a composite number, instead of the number itself, see § 19. 3.

3. *Examples.*

(1.) Multiply 4821 by 742.	*Ans.* 3577182.
(2.) Multiply 3658 by 848.	*Ans.* 3101984.
(3.) Multiply 53168 by 927.	*Ans.* 49286736.
(4.) Multiply 89467 by 279.	*Ans.* 24961293.
(5.) Multiply 621458 by 545.	*Ans.* 338694610.
(6.) Multiply 468921 by 455.	*Ans.* 213359055.
(7.) Multiply 796124 by 428.	*Ans.* 340741072.
(8.) Multiply 246101 by 639.	*Ans.* 157258539.
(9.) Multiply 468257 by 2763.	*Ans.* 1293794091.
(10.) Multiply 331458 by 85672.	*Ans.* 28396669776.

§ 26. 1. CASE IX. *To multiply by* 1, *with either of the digits annexed to it; as,* 11, 12, 13, 14, *&c.*

RULE. Indicate the multiplication by the sign; multiply by the right hand figure of the multiplier; write the product under the multiplicand, one place to the right; add it to the multiplicand, and the sum will be the required product.

2. *Illustration.*

```
3754×19.          3754×19
                  33786
                  -----
                  71326
```

3. *Explanation.* Writing the product of the units one place to the right, leaves the multiplicand as the product

of the *ten*, with its first figure in the *ten's* place, and so, in effect, multiplies it by 10. It is as though the multiplicand were first multiplied by the *ten;* the cipher being kept in mind, not expressed; and then by the units.

4. *Examples,*

(1.) Multiply 734 by 12. *Ans.* 8808.
(2.) Multiply 4643 by 13. *Ans.* 60359.
(3.) Multiply 5356 by 17. *Ans.* 91052.
(4.) Multiply 63412 by 19. *Ans.* 1204828.
(5.) Multiply 71589 by 15. *Ans.* 1073835.
(6.) Multiply 93567 by 18. *Ans.* 1684206.
(7.) Multiply 82456 by 14. *Ans.* 1154384.
(8.) Multiply 58934 by 16. *Ans.* 942944.

(9.) If there are 18 panes of glass in a window, how many are there in a house which has 52 such windows? *Ans.* 936.

(10.) If 19 men can do a piece of work in 89 days, how many men can do it in one day? *Ans.* 1691.

(11.) There is a certain number, the factors of which are 17 and 462. What is the number? *Ans.* 7854.

(12.) A farmer sold 375 pounds of pork, at 14 cents a pound; and 148 pounds of cheese, at 13 cents a pound. How many cents did he receive in pay? *Ans.* 7174 cents.

§ 27. 1. CASE X. *To multiply by* 1, *with either of the digits prefixed to it; as,* 21, 31, 41, 51, *&c.*

RULE. Indicate the multiplication by the sign; multiply by the left hand figure of the multiplier; write the product under the multiplicand, one place to the left; add it to the multiplicand, and the sum will be the required product.

2. *Illustration,*

$$5684\times21. \qquad \begin{array}{r} 5684\times21 \\ 11368 \\ \hline 119364 \end{array}$$

3. *Explanation.* This method differs from Case III.

(§ 20.), only in its regarding the multiplicand as the product by the unit, and so avoiding its repetition.

4. *Examples.*

(1.) Multiply 13864 by 71. *Ans.* 984344.
(2.) Multiply 73918 by 21. *Ans.* 1552278.
(3.) Multiply 2468 by 91. *Ans.* 224588.
(4.) Multiply 482436 by 81. *Ans.* 39077316.
(5.) Multiply 91738 by 81. *Ans.* 7430778.
(6.) There are 41 pieces of cloth, containing each 112 yards; how many yards are there in all? *Ans.* 4592 yards.
(7.) Suppose a man travel by steam 2190 miles in a year; how far will he travel in 61 years? *Ans.* 133590 miles.
(8.) How many hills are there in a field of corn, containing 149 rows, with 71 hills in a row? *Ans.* 10579 hills.
(9.) There are 365 days in a year; how many days are there in 21 years? *Ans.* 7665 days.

§ 28. 1. CASE XI. *To multiply as in the last two cases, when ciphers intervene between the significant figures.*

RULE. Proceed as in Cases IX. or X., according as the ciphers are preceded by 1, or by a higher figure; if by 1, according to Case IX.; if by a higher figure, according to Case X.; taking care to write each figure in the product in its proper place (§ 26. and § 27).

2. *Illustrations.*

(1st.) 3564×109.

```
3564×109
 32076
------
388476
```

(2d.) 2978×901.

```
2978×901
 26802
-------
2683178
```

(3d.) 123581×832001.

```
    123581×832001
988648
 3954592
------------
102819515581
```

(1.) In the first illustration, the product by the units is written two places to the right, in order to leave the multiplicand, as the product by 1 *hundred*, with its first figure in the hundreds' place.

(2.) In the second illustration, the product by the hundreds is removed two places to the left, to give it its proper value.

(3.) In the third illustration, the 32 is a multiple of 8 and 4. We first multiply by the 8, the factor of the multiple expressed, and that product by 4, the other factor in the multiple, (see Case VII.), and write the products as before directed and explained (§ 25).

8. *Examples.*

(1.) Multiply 7598 by 104. *Ans.* 790192.
(2.) Multiply 6735 by 1005. *Ans.* 6768675.
(3.) Multiply 56934 by 301. *Ans.* 17137134.
(4.) Multiply 37562 by 601. *Ans.* 22574762.
(5.) Multiply 6456 by 105. *Ans.* 677880.
(6.) Multiply 344620 by 108. *Ans.* 37218960.
(7.) Multiply 34567 by 4001. *Ans.* 138302567.
(8.) How many letters are there in a volume of 742 pages, each page containing 401 lines, and each line 109 letters? *Ans.* 32432078.
(9.) There are 24 hours in a day; how many hours are there in 108 days? *Ans.* 2592.

§ 29. 1. The accuracy of an operation may be tested by multiplying the multiplier by the multiplicand. If the products agree, there is probably no error; as we should not be likely to commit the same error multiplying two different ways.

2. *Illustration.* 441×23.

```
  441        23
   23       441
 ----      ----
 1323        23
 882        92
 -----     92
 10143     -----
           10143
```

3. Or, as many times the multiplicand may be added together, as there are units in the multiplier.

4. *Illustration.* 44×3=132

$$\begin{array}{r} 44 \\ 44 \\ 44 \\ \hline 132 \end{array}$$

§ 30. 1. *Proof.** Multiplication, like Addition, may be proved by Subtraction; that is, by subtracting either factor from the product, and the same from that remainder, and so on, till nothing remains; then, if the number of the subtractions equal the other factor, the operation is correct.

2. Or, shorter, by Division; that is, by dividing the product by either factor. If the quotient be the other factor, the operation is correct.

3. *Illustrations.*

(1.) 6×5=30

$$\begin{array}{rl} 30 & \\ 6 & 1. \\ \hline 24 & \\ 6 & 2. \\ \hline 18 & \\ 6 & 3. \\ \hline 12 & \\ 6 & 4. \\ \hline 6 & \\ 6 & 5.\text{=the other factor.} \\ \hline 0 & \end{array}$$

(2.) 8×6=48

$$\begin{array}{r} 8)48 \\ \hline 6 \end{array}$$

4. *Explanation.* The explanation of the proof for Addition, is sufficient for the first method above (§ 16. 5, 7); and that for the principle of Division (§ 35. 4), for the other.

* This proof should be passed over, till the pupil has become familiar with Subtraction and Division.

V. SUBTRACTION.

§ 31. 1. SUBTRACTION is the taking of one number from another; or the separating of a number into two others, one of which is given. Thus, 8 may be taken from 18; and we shall have, as the equivalent of the 18, the two numbers 8 (that which was given) and 10.

2. It is, therefore, directly the reverse of Addition, or the bringing of numbers together (§ 15. 1).

3. The number from which the subtraction is made, is called the *minuend;* that subtracted, the *subtrahend;* the result, the *remainder.*

4. Subtraction is indicated by a horizontal line (—), called *minus*, signifying less. 4—3, denotes 4 less 3, or that 3 is to be subtracted from 4.

§ 32. 1. CASE I. *To subtract, when each figure in the minuend equals or exceeds the corresponding figures in the subtrahend.*

RULE. I. Write the subtrahend under the minuend, in the order of, units under units, tens under tens, and so on; and draw a horizontal line underneath.

II. Beginning at the right, take each figure in the subtrahend from that above it, and write the difference underneath; or, when there is no difference, write a cipher.

2. *Illustration.* 9643187—7532043.

(1.) We write the minuend, the number from which the subtraction is to be made, first, and the subtrahend, the number to be subtracted, under it, in the order of units under units, tens under tens, and so on; and draw a horizontal line under them.

```
9643187 Minuend.
7532043 Subtrahend.
-------
2111144 Remainder.
```

(2.) We then take the difference between each of the

corresponding figures, beginning at the right, with the units, saying 3 from 7 equals 4, and write the difference directly under these figures; and so on to the last.

3. *Explanation.* The subtrahend is placed under the minuend for convenience; and, in this case, the subtraction is commenced at the right, for the same reason. The numbers we have taken might be resolved into the parts which compose them, and then the principle of the operation would be yet more apparent.

9643187=9000000+600000+40000+3000+100+80+7
7532043=7000000+500000+30000+2000+000+40+3

2000000+100000+10000+1000+100+40+4

2000000
100000
10000
1000
100
40
4

2111144=the Remainder, in the Illustration.

4. *Examples.*

(1.) From 9816372 take 2713252. *Ans.* 7103120.
(2.) From 49384750 take 41273520. *Ans.* 8111230.
(3.) From 56789432 take 12345322. *Ans.* 44444110.
(4.) From 81726354 take 50615241. *Ans.* 31111113.
(5.) There are two farms; one is valued at 2750, and the other at 1500 dollars; what is the difference in their value? *Ans.* 1250 dollars.
(6.) A person owed a merchant 999 guineas, and paid him all but 179. How many did he pay him? *Ans.* 820 guineas.
(7.) If the sum of two numbers be 2793, and one of those numbers 1692, what is the other? *Ans.* 1101.
(8.) A merchant bought a quantity of cloth for 575 dollars, and sold it for 523 dollars. Did he gain or lose, and how much? *Ans.* Lost, 52 dollars.

(9.) Gunpowder was invented in the year 1330; how long was this before the invention of printing, which was in 1441? *Ans.* 111 years.

§ 33. 1. CASE II. *To subtract, when some figures in the minuend are less than the corresponding figures in the subtrahend.*

RULE. I. Write down the numbers as in Case I.

II. Beginning at the right, subtract, when it can be done, as in Case I.; but, when any figure in the minuend is less than the corresponding figure in the subtrahend, suppose 1 to be taken from the figure next above it, to stand at its left, and be added to it.

III. From this sum subtract, and write the difference underneath.

IV. If the figure next above, in the minuend, be cipher, regard it as 10, by supposing 1 taken from the figure next above it, to stand at its left, and to be added to it.

V. In subtracting from the figure next above in the minuend, regard it as diminished 1, in consequence of the 1 taken from it to increase the figure at the right.

2. *Illustration.* 600198056—356499278.

(1.) We write the subtrahend under the minuend, units under units, tens under tens, and so on; and draw a horizontal line underneath.

$$\begin{array}{r} 600198056 \\ 356499278 \\ \hline 243698778 \end{array}$$

(2.) We begin at the right, and endeavor to subtract the units of the subtrahend from those of the minuend; but we find the minuend the less; 8 cannot be taken from 6. We therefore, suppose 1 taken from the 5, the next higher figure in the minuend, which is a ten, and therefore equal to 10 units, to stand at the left of the 6, and to be added to it, making 16. We then say 8 from 16 leaves 8, to be written in the place of units.

(3.) In the second place, or that of the tens, we have then only a 4, instead of a 5; and as 7 is to be subtracted, we are under the necessity of taking 1 from the next higher figure, to increase it by 10. This being 0, it is made 10, by taking 1 from 8, the figure next above it, and placing it at its left, and adding it to it. Subtracting, then, 7 from 14, the remainder, 7, is written in the proper place.

(4.) In the next place, the place of hundreds, we have then a 9, 1 having been taken from it, when it was made 10, and used in subtracting the tens, from which 2 being subtracted, gives the remainder, 7.

(5.) In the next place, the 8 has become 7, by the process of making the 0 in the next lower place 10, and is insufficient to admit of a 9 being subtracted from it. It is then increased by 1 from the figure next above it, and so made 17, when the 9 is subtracted, and leaves 8 to be written down.

(6.) So we proceed with the other figures to the 6, from which, it being larger than the corresponding figure in the subtrahend, the 3, the subtraction is made.

(7.) We marked every figure in the minuend from which 1 was taken, by a dot above it, in order that it might not be forgotten, when the subtraction was to be made from it, that it had been diminished. It will be well for beginners to do this till they have become perfectly familiar with the operation.

3. *Explanation.* The fact that Subtraction is the reverse of Addition (§ 31. 2), of itself explains the propriety of taking 1 from the next higher figure in the minuend, when the subtraction cannot be made from the lower, and using it with that lower figure.

In a directly reverse operation, it is evident that each principal step should be reversed. As, then, in bringing numbers together, ten at the right, gives 1 to the left, to the next higher column (§ 16. 3); so in taking numbers apart, 1 at the left, gives ten to the right, to the next lower column.

§ 34. 1. *Proof.* Subtraction is proved by reversing the operation; by bringing together again the numbers taken apart; that is, by addition of the Subtrahend and Remainder. If their sum equals the minuend, the operation is correct (§ 16. 7).

2. *Illustration.*

$$\begin{array}{rl} 536872 & \\ 397218 & \text{Subtrahend.} \\ \hline 139654 & \text{Remainder.} \\ \hline 536872 & = \text{the Minuend.} \end{array}$$

§. 35. *Examples.*

(1.) From 67143 take 41872. *Ans.* 25271.
(2.) From 467328 take 179814. *Ans.* 287514.
(3.) From 2637804 take 2376982. *Ans.* 260822.
(4.) From 8592678 take 1078953. *Ans.* 7513725.
(5.) From 3762162 take 826541. *Ans.* 2935621.

(6.) From thirty thousand and ninety-seven, take one thousand six hundred and fifty-four. *Ans.* 28443.

(7.) From one million, take one. *Ans.* 999999.

(8.) From 1600000 take 900000; and from that remainder take 699999; and how much will remain? *Ans.* 1.

(9.) A farmer purchased a farm, for which, including the buildings, he paid 6782 dollars; the buildings were worth 2896 dollars. What was the value of the land? *Ans.* 3886 dollars.

(10.) The birth of the Saviour took place in the year of the world 4004; the flood of Noah about 2348 years before that event. How long was it from the creation to the flood? *Ans.* 1656 years.

(11.) The call of Abraham took place about 1921 years before Christ. How many years was that after the creation? *Ans.* 2083 years.

(12.) Solomon finished the temple about the year of the world 3989. How many years have elapsed since that event? *Ans.* 2856 years.

(13.) Isaiah's prophecy was concluded about the year of the world 3306. How many years were there from that to the birth of Christ? And how many have elapsed since his prophecy was given?
Ans. To the birth of Christ, 698 years.
To this time, 2539 years.

(14.) The first book ever printed, with types, was a copy of the Bible, about the year 1451. How many years have elapsed since that event? *Ans.* 390 years.

VI. DIVISION.

§ 36. 1. DIVISION is the separating of one number into as many parts as is denoted by another; or the taking of one number from another a certain number of times.

2. It is not an essentially distinct operation, but is simply a convenient and concise method of performing Subtraction. Thus, if it were required to separate 18 into parts containing 6 each, the operation, by subtraction, would be to subtract the 6 from the 18, and then again from that remainder, and again from that, when there would be no remainder. (1.) 18—6=12. (2.) 12—6=6. (3.) 6—6=0. Then the sum of the subtractions, 3, would be the answer; that is, would show into how many parts, of 6 each, 18 might be separated.

3. By Division, we should at once see into how many times 6 parts, the 18 might be divided. The numbers would be but once written, with the result underneath, or at the right. Thus, 6)18(3.
It is evident, that two of these numbers, that denoting into how many parts the separation is to be made and the result, correspond precisely to the *factors* producing the product in multiplication; and that the number to be separated, corresponds precisely to the product (§ 17. 3).

4. In Division, therefore, we have a product and one factor given, to obtain the other factor.

5. The number to be divided is called the *dividend;* that denoting the parts into which the division is to be made, the *divisor;* the result, the *quotient.*

6. Division is indicated in four ways; by a sign, consisting of a horizontal line, with a dot above and below it (÷), called the sign of division; by writing the dividend at the right of the divisor, with a curved line between them ( )); by writing the divisor under the dividend, with a horizontal line between them (—); and by two dots, the colon (:), between the divisor and dividend. 18÷6; 6)18; $\frac{18}{6}$; 6:18; all signify the same thing, that 18 is to be divided by 6.

§ 37. 1. CASE I. *To divide by a single figure.*

RULE. I. Write the divisor at the left of the dividend, with a curved line between them, and draw a horizontal line underneath.

II. Find how many times the divisor will divide the first or left hand figure in the dividend, if that will contain it; if not, the first two left hand figures, and write the result directly under, for the first figure of the quotient.

III. Multiply, in mind, this quotient figure by the divisor, and subtract the result from the first figure or figures in the dividend.

IV. Suppose the remainder, if there be any, to stand at the left of the next figure at the right in the dividend, and to be added to it.

V. Find how many times the divisor is contained in this sum; and so proceed to the last.

VI. If then there be a remainder, write it at the right of the quotient, with the divisor underneath, and a horizontal line between them.

2. *Illustration.* 8421÷3.

```
3)8421
  ----
  2807
```

(1.) Here we say, 3 in 8 is contained twice; and having written the 2 under the 8, as the first figure of the quotient, we, in mind, multiply it by the divisor, and subtract the result, 6, from the first figure of the dividend, the 8, which leaves 2 as a remainder. We then suppose this remainder, 2, to stand at the left of the next figure at the right in the dividend, 4, and to be added to it, which gives 24. In this sum, 24, we find the divisor is contained 8 times; writing the 8 in the quotient, we multiply it by the divisor, which gives the result, 24.

(2.) We then subtract the result from the sum, which being also 24, leaves no remainder.

(3.) We now proceed to the next figure in the dividend, 2, in which we find the divisor is not contained. In the quotient, we place a 0, to preserve the value of the figures in the quotient above it, and suppose the 2 to stand at the left of 1, the next figure at the right in the dividend, and

to be added to it. In this sum, 21, we find the divisor is contained 7 times; writing the 7 in the quotient, we multiply it by the divisor, which gives the result, 21.

(4.) This result we subtract from the sum, which being also 21, leaves no remainder. We thus obtain the exact quotient, 2807.

(5.) In dividing figures of any local value, by simple units, it is obvious the quotient figure or figures must have a local value to correspond with that of the figure or figures divided. Thus, 2 in the quotient, occupies the place of hundreds of thousands, to correspond with the place of 8, the figure divided. The division, then, is that of 8 hundreds of thousands, into three parts, or by 3. The result, of course, must be hundreds of thousands; and this is expressed by the 2 in that place. So of the rest.

3. Again; 7835921÷8.

$$\begin{array}{r} 8)\underline{7835921} \\ 979490\frac{1}{8} \end{array}$$

(1.) Here we find the first left hand figure, 7, will not contain the divisor; we must, therefore, take it in connection with the next figure to the right, 8. In 78 we find the divisor is contained 9 times; having written the 9 under 78, as the first figure of the quotient, we, in mind, multiply it by the divisor, and subtract the result from the 78, which leaves 6.

(2.) This 6 we suppose to stand at the left of the next figure at the right in the dividend, and to be added to it; and so proceed with the operation exactly as in the last illustration, until we come to the last figure in the dividend, 1, in which the divisor is not contained; under it, therefore, we write 0.

(3.) Having no more figures in the dividend, we find that 1 ought yet to be divided by 8; we then write in the quotient, $\frac{1}{8}$, which signifies that over and above the even parts denoted by the preceding figures, there is yet 1 to be divided by 8; a division, which in this stage of the subject, we are not prepared to execute. We, therefore, thus have it as an unexecuted division.

(4.) This is known as Short Division.

4. *Explanation.* While all the preceding operations begin at the right, with units, this, on the contrary, begins at the left, with the highest figure. The propriety of this will at once appear, when it is recollected that this operation corresponds precisely to the undoing of an addition, as illustrated (§ 15. 6).

(1.) The dividend regarded as a *product* (§ 36. 4), corresponds to the *sum* of so many times one factor as there are units in the other. Taking up the dividend, then, as a sum, we would separate it precisely as we would the sum of an addition to prove its correctness. That is done by beginning at the left, taking out of the highest figure or figures in the sum, the sum of the highest column, subtracting it from the sum first obtained, and supposing the remainder to stand at the left of the next figure in the sum, and so on (§ 15. 6).

(2.) Each remainder so taken to the right, is produced from what, in the supposed addition, or multiplication, of the numbers producing the dividend, was taken to the left. It is obvious, then, that the separation must begin at the left, with the highest figures, in order that those parts which were taken to the left, in producing the dividend, may be restored to the right, and be separated with the inferior, or right hand figures, with which they belong, and from which they were carried to the left.

5. *Examples.*

(1.) Divide 635 by 5. Ans. 127.
(2.) Divide 5463 by 3. Ans. 1821.
(3.) Divide 1512 by 7. Ans. 216.
(4.) Divide 1256 by 4. Ans. 314.
(5.) Divide 7368 by 2. Ans. 3684.
(6.) Divide 598680 by 8. Ans. 74835.
(7.) Divide 4634891 by 9. Ans. $514987\frac{8}{9}$.
(8.) Nine persons draw a prize of 198 dollars. What was each man's share? Ans. 22 dollars.
(9.) A person dying, leaves an estate of 4515 dollars to 5 children. What will be each one's share? Ans. 903 dollars.
(10.) Four persons boarded at a public house till the bill of their board was 368 dollars. What was the average bill? Ans. 92 dollars.

(11.) How many times may 9 be subtracted from 675? Ans. 75.

(12.) If a man traveled 441 miles in 7 days, what was his daily progress? Ans. 63 miles.

§ 38. 1. CASE II. *To divide by* 10, 100, 1000, *or by* 1, *with any number of ciphers annexed.*

RULE. Point off, or suppose to be pointed off, by a comma, from the right of the dividend, as many figures as there are ciphers in the divisor. The figures thus pointed off, are a remainder, under which write the divisor. The figures at the left and that remainder, are the quotient.

2. *Illustrations.*

(1.) 48750÷10. 10)4875,0 or, 48750÷10=4875

(2.) 35600÷100. 100)356,00 or, 35600÷100=356

(3.) 765÷10. 10)76,$\frac{5}{10}$ or, 765÷10=76$\frac{5}{10}$

3. *Explanation.* Pointing off a figure from the right of the dividend, causes the units' place to be the remove of one figure to the left, and so of the others; and so, by diminishing the dividend, gives us one of ten parts of it.

4. The remainder is a division, which in this stage of the subject, cannot be executed.

5. *Examples.*

(1.) Divide 7834 by 10. Ans. 783$\frac{4}{10}$.

(2.) Divide 72456 by 10. Ans. 7245$\frac{6}{10}$.

(3.) Divide 64289 by 100. Ans. 642$\frac{89}{100}$.

(4.) Divide 812567 by 1000. Ans. 812$\frac{567}{1000}$.

(5.) Divide 9416700 by 100. Ans. 94167.

(6.) Divide 50100 by 10. Ans. 5010.

(7.) In one dollar there are 100 cents. How many dollars are there in 4567840 cents?
Ans. 45678$\frac{40}{100}$ dollars.

(8.) In one cent there are 10 mills. How many cents are there in 37211 mills? Ans. 3721$\frac{1}{10}$ cents.

(9.) In one dollar there are 1000 mills. How many dollars in 83467800 mills? Ans. 83467$\frac{800}{1000}$ dollars.

(10.) If a tax of 40000 dollars were to be levied upon 1000 polls, what would each poll pay? Ans. 40 dollars.

§ 39. 1. Case III. *To divide by two or more significant figures.*

Rule. I. Write the divisor at the left of the dividend, with a curved line between them; and draw a similar line at the right.

II. Take, in mind, as many of the left hand figures as will contain the divisor once or more; see how many times the divisor is contained in them, and write the result, at the right of the dividend, for the first figure of the quotient.

III. Multiply the divisor by this quotient figure; write the product under those figures of the dividend which gave it, and subtract the product from the figures above it.

IV. To the remainder, annex the next figure of the dividend; again divide, if it can be done; if not, write cipher in the quotient, and annex another figure in the same way; then divide, write the result in the quotient, and multiply, &c., as before.

V. In this way proceed to the last.

VI. If then there be a remainder, write the divisor under it, at the right of the figures in the quotient, and they, with it, will be the quotient required.

2. *Illustration.*

$$64059 \div 758.$$

$$\begin{array}{r} 758)64059(84\frac{387}{758} \\ 6064\phantom{9(84\frac{387}{758}} \\ \hline 3419\phantom{(84\frac{387}{758}} \\ 3032\phantom{(84\frac{387}{758}} \\ \hline 387\phantom{(84\frac{387}{758}} \end{array}$$

(1.) Here we find the least number of figures that will contain the divisor is four, and in them we find the divisor is contained 8 times, which we write at the right of the dividend for the first figure of the quotient.

(2.) We then multiply the divisor by this quotient, writing the product, 6064, under the figures of the dividend

in which the divisor was contained, 6405, and subtract the product from the figures above it.

(3.) We then have a remainder, 341, which, being less than the divisor, shows that no greater number could have been taken as the quotient. To this remainder, we annex the next figure of the dividend, 9, which gives the number, 3419.

(4.) In this number we find the divisor is contained 4 times, which we write at the right of the 8 already obtained in the quotient.

(5.) As before, we multiply the divisor by this quotient, writing the product, 3032, under 3419, and subtract the product from the figures above it.

(6.) We again have a remainder, 387, which is less than the divisor, and which must be divided. Having no more figures in the dividend to annex to it, we write it over the divisor, at the right of the quotient, $84\frac{387}{758}$, as in the second illustration in Case I. (§ 37. 3).

3. *Explanation.* All the principles in this operation are the same as those involved in Case I. The exposition of them there, therefore, is sufficient.

4. This detailed mode of operating, is called Long Division.

5. *Examples.*

(1.) Divide 7210473 by 37. Ans. $194873\frac{24}{37}$.
(2.) Divide 147735 by 45. Ans. 3283.
(3.) Divide 937387 by 54. Ans. $17359\frac{1}{54}$.
(4.) Divide 145260 by 108. Ans. 1345.
(5.) Divide 1203033 by 3679. Ans. 327.
(6.) Divide 74855092410 by 949998. Ans. 78795.

(7.) The product is 68959488; the multiplier, 96; what is the multiplicand? Ans. 718328.

(8.) The sum of 19125 dollars is to be distributed among a certain number of men; each is to receive 425 dollars. How many men are to receive the money? Ans. 45.

(9.) If a man walks 12775 miles in a year, or 365 days, how far does he walk each day? Ans. 35 miles.

(10.) A person goes to a store and buys a piece of cloth containing 36 yards, for which he pays 288 dollars. How much does he pay a yard? Ans. 8 dollars.

(11.) The provision of an army, in bread, is 90567 lbs.; it is intended to distribute the whole to the soldiers, to save separate transportation; there are 10063 soldiers; how many pounds will each have to carry? Ans. 9 lbs.

(12.) The expenses of paving a street, 500 feet in length, amount to $1000; the amount is to be distributed among the owners of the adjoining lots, each having a lot of 25 feet; how much will each lot or owner have to pay?
Ans. 50 dollars.

(13.) What number must you multiply by 47, to produce 298804098? Ans. 6357534.

6. Or, as it is often desired to spare writing out the numbers for the subtraction, writing down only the remainders, we, therefore, have;

§ 40. 1. Case IV. *To divide, by two or more figures, writing down only the quotient and remainders.*

Rule. Divide as in Case III.; perform, in mind, the multiplications and subtractions, and write down only the remainders.

2. *Illustration.*

9460753÷879. 879)9460753(10763$\frac{76}{879}$
6707
5545
2713
76

(1.) Here the divisor is contained once in the first three figures of the dividend; writing the 1 in the quotient, we multiply and subtract, in memory, writing only the remainder below; which is 76.

(2.) The 0, the next figure of the dividend, being annexed to this remainder, makes it 670, which being less than the divisor, 879, the next figure for the quotient is 0 (§ 39. 1. Rule. IV.).

(3.) After writing 0 in the quotient, the next figure in the dividend is annexed to the remainder; in the result, 6707, we find the divisor is contained 7 times. This 7 we write in the quotient; multiply each figure in the divisor by it, in memory; subtract the product of each as we proceed, in

the same way, and write down only the remainder. So we proceed to the last, where we have a remainder, 76, which we write as in the preceding Case (§ 39. 1. Rule. VI.).

3. *Explanation.* This operation is precisely similar to that in Case III., except that we there write down what we here bear in mind.

The exposition of the principle of division, made under Case I., is sufficient for all that follows it, in division, this included.

4. *Examples.*

(1.) Divide 46242 by 252. Ans. $183\frac{126}{252}$.
(2.) Divide 45817 by 35. Ans. $1309\frac{2}{35}$.
(3.) Divide 1198714 by 396. Ans. $3027\frac{22}{396}$.
(4.) Divide 2019126 by 1918. Ans. $1052\frac{1390}{1918}$.
(5.) Divide 1091462 by 5939. Ans. $183\frac{4625}{5939}$.
(6.) Divide 1896371 by 119. Ans. $15935\frac{106}{119}$.
(7.) A field of 34 acres produced 1020 bushels of corn. How much was that per acre? Ans. 30 bushels.
(8.) A privateer of 175 men took a prize worth 20650 dollars, of which the owner of the privateer had one half, and the rest was equally divided among the men. What was each man's share? Ans. 59 dollars.
(9.) A certain township contains 30000 acres. How many lots, of 125 acres each, does it contain? Ans. 240.
(10.) If a certain number of men, by paying 33 dollars each, paid 726 dollars, what was the number of the men? Ans. 22.

§ 41. 1. Case IV. *To divide, when there are ciphers at the right of the divisor.*

Rule. I. Write the numbers as in Case III.

II. Point off from the right of the dividend as many figures as there are ciphers at the right of the divisor; divide by the figures at the left of the ciphers only; annex the figures cut off, to the remainder, if there be any, and write under them the divisor, at the right of the figures in the quotient, and they together will form the quotient required.

2. *Illustrations.*

(1.) $43790 \div 90$. $\quad 9,0)4379,0$ or, $43790 \div 90 = 486\frac{50}{90}$

$\quad\quad\quad\quad\quad\quad \overline{486\frac{50}{90}}$

(2.) $6549 \div 340$.

```
34,0)654,9(19 89/340
     34
     ---
     314
     306
     ---
      89
```

(3.) $6466 \div 3200$.

```
32,00)64,66(2 66/3200
      64
      ---
       66
```

3. *Explanation.* The effect of pointing off the ciphers is explained in Case II.; and the illustrations and explanations to Cases I. and III., are sufficient for the remainder of the operation.

4. *Examples.*

(1.) Divide 876000 by 6000. Ans. 146.
(2.) Divide 432132 by 9200. Ans. $46\frac{8932}{9200}$.
(3.) Divide 67389 by 700. Ans. $96\frac{189}{700}$.
(4.) Divide 35397 by 150. Ans. $569\frac{47}{150}$.
(5.) Divide 146340 by 5400. Ans. $27\frac{540}{5400}$.
(6.) Divide 741725 by 900. Ans. $824\frac{125}{900}$.

(7.) A merchant has 5100 pounds of tea, and wishes to pack it in 60 chests. How many pounds must he put in each chest? Ans. 85.

(8.) How many yards of cloth can be bought for 3460 dollars, at 20 dollars a yard? Ans. 173.

(9.) If 10500 dollars are given for 750 barrels of flour, I demand the price of each barrel. Ans. 14 dollars.

(10.) If 40 horses were sold in the West Indies for 9900 dollars, what was the average price? Ans. $247\frac{20}{40}$ dollars.

(11.) Sixty men, at a feast, spend 240 dollars. How much does each man spend? Ans. 4 dollars.

§ 42. 1. CASE V. *To divide by a composite number* (§ 17. 4).

RULE. I. Divide the given dividend first by one of the factors, and that quotient by the other, and the last quotient will be the answer.

II. If there be a remainder after each division, write it at the right; and, when the division is performed, to find the true remainder, multiply the first divisor and last remainder together, and to the product add the first remainder.

2. *Illustration.* $182641 \div 72$.

$9 \times 8 = 72$.	9)182641		First divisor,	9
	8)20293	4	Last remainder,	5
				45
	2536	5	First remainder,	4
True Ans. $2536\frac{49}{72}$.			True remainder=	49

3. *Explanation.* (1.) The principle on which the multiplication, by a composite number, was made, is sufficient for this division (§ 19. 3).

(2.) The principle on which the true remainder is obtained, is as follows:*

The first remainder is so much of the dividend undivided; and, therefore, may be regarded as the numerator of a fraction, of which the first divisor is the denominator.

The second remainder, like the second dividend, is in parts, such as are denoted by the first divisor.

To obtain the true remainder, then, in one, these, of course, must be united. The last remainder is, therefore, changed to parts, such as are denoted by the first divisor, by multiplying it by that divisor.

The two remainders, then, are fractions, with the first divisor as a common denominator to each. These fractions are then added; and their sum is divided by the other factor in the whole divisor; which gives a fraction

* This part of the explanation should be passed over, till the pupil has become familiar with Vulgar Fractions.

to be divided by a whole number. To effect this, we multiply the denominator by this number, and obtain the whole divisor as the denominator of the fraction. This fraction is the true remainder.

4. *Examples.*

(1.) Divide 2592 by 63. Ans. $41\frac{9}{63}$.
(2.) Divide 4688 by 48. Ans. $97\frac{32}{48}$.
(3.) Divide 6750 by 15. Ans. 450.
(4.) Divide 60400 by 25. Ans. 2416.
(5.) Divide 45678 by 16. Ans. $2854\frac{14}{16}$.
(6.) Divide 786493 by 81. Ans. $9709\frac{64}{81}$.
(7.) Divide 328764 by 72. Ans. $4566\frac{12}{72}$.
(8.) If a horse run 288 miles in 36 hours, how far does he run in one hour? Ans. 8 miles.
(9.) In 437850 yards of cloth, how many rolls of 75 yards each? Ans. 5838.
(10.) If 30114 dollars be divided equally among 63 men, how many dollars will each receive? Ans. 478.

§ 43. 1. *Proof.* Division is proved by reversing the operation; that is, by multiplying together the *divisor* and *quotient;* and, if there be a remainder, adding the remainder to the product (§ 36. 1. § 34. 1).

2. If the operation be correct, this result will correspond with the dividend (§ 36. 3, 4).

3. *Illustration.*

(2.) 758)64059($84\frac{387}{758}$	758	Divisor.
6064	$84\frac{387}{758}$	Quotient.
3419	3032	
3032	6064	
387	387	Remainder.
	64059=Dividend.	

4. *Explanation.* The *dividend* is regarded as a product, of which the divisor and quotient are factors (§ 36. 3), and the remainder a part. The multiplication of the factors, and the addition of the part, must, of course, give the product, or, in this case, the dividend.

§ 44. 1. *Miscellaneous Examples, involving all the preceding operations.*

(1.) A merchant bought 56 bales of cotton goods; 15 of them held 21 pieces, 29 held 28 pieces, and the rest 25 pieces each; for each piece he agreed to pay 3 dollars; how much had he to pay for the whole?

Ans. 4281 dollars.

(2.) The difference of two numbers is 723, and their sum is 1111, what are the numbers?

Ans. $\begin{cases} 194. \\ 917. \end{cases}$

(3.) If I plant in 14 rows, 2072 fruit trees, and set the trees 25 feet asunder, how many feet long are the rows?

Ans. 3675 feet.

(4.) If the divisor be 32, and the quotient 204, what is the dividend? Ans. 6528.

(5.) A farmer sells 4 tons of hay at 12 dollars a ton; 80 bushels of wheat at 1 dollar a bushel, and takes, in part payment, a horse worth 65 dollars, a wagon worth 40 dollars, and the rest in cash. How much money did he receive? Ans. 23 dollars.

(6.) A man has 6 children, all of whom are married, and each has 4 children; 2 of these grand children are married, and each has 1 child. How many children, grand children and great grand children are there. Ans. 32.

(7.) Rome being built 817 years after the Pyramids of Egypt, and 146 years before the Christian Era, America having been discovered in 1594, the Declaration of Independence by the United States having taken place 182 years afterwards, and the year of it now counted being 65; how many years since the building of the Egyptian Pyramids? Ans. 2804 years.

(8.) Subtract 30079 from ninety-three millions as often as it can be done, and say how much the last remainder exceeds or falls short of 21180? Ans. 4631 exceeds.

(9.) The least of two numbers is 77, and their difference is 99, what is the greater? Ans. 176.

(10.) There are four numbers; the first, 317; the second, 912; the third, 1229; and the fourth as much as the other three, abating 97; what is the sum of them all?

Ans. 4819.

(11.) A gentleman left his son 1725 pounds more than

his daughter, whose fortune was 15 thousand, 15 hundred and 15 pounds; what was the son's portion, and what did the whole estate amount to?

Ans. { The son's portion, £18240, and the estate, £34755.

(12.) What number, deducted from the 32d part of 3072, will leave the 96th part of the same? Ans. 64.

(13.) What is the difference between thrice five, and thirty, and thrice thirty-five? Ans. 60.

(14.) Suppose the quotient arising from the division of two numbers to be 5379, the divisor, 37625; what is the dividend, if the remainder come out 9357?

Ans. 202394232.

(15.) A privateer of 175 men took a prize which amounted to 59 pounds a man, besides the owner's half; what was the value of the prize? Ans. £20650.

(16.) There are two numbers; the greater of them is 25 times 78, and their difference is 9 times 15; their sum and product is required.

Ans. 1950 the greater; 1815 the less; 3765 the sum; and 3539250 the product.

(17.) A merchant began trade with 25327 dollars; for 6 years together, he cleared 1253 dollars per annum; the next 5 years, he cleared 1729 dollars per annum; but, the last 4 years, he had the misfortune to lose 3019 dollars per annum; what was he worth at the 15 years' end?

Ans. 29414 dollars.

(18.) If 9000 men march in a column of 750 deep, how many march abreast? Ans. 12.

(19.) The least of two numbers is 19418, and the difference between them is 2384; what is the greater, and sum of both?

Ans. 21802 the greater, and 41220 the sum.

(20.) What number added to the 27th part of 6615, will make 570? Ans. 325.

(21.) What is the difference betwen six dozen dozen, and half a dozen dozen; and what is their product, and the quotient of the greater by the less?

Ans. 792 difference, 62208 product, and 12 quotient.

(22.) Divide 151200 lbs. of beef equally among an army, consisting of 27 regiments, each regiment 7 companies, and each company 100 men. Ans. 8 lbs.

PART II.

FRACTIONS.

§ 45. 1. In Division, at the close of an operation, we frequently had a remainder, which, being smaller than the divisor, we were unable to divide, with the means then possessed (§ 37. 3, (3). The quotient, however, being incomplete without that division, we indicated the actual result, and so completed the quotient, by writing the divisor under the remainder, with a horizontal line between them, at the right of the quotient figures previously obtained (§ 37. 3).

2. Such expressions have given rise to a distinct class, or modification of numbers, called *Vulgar Fractions*.

3. The origin of them, thus given, constantly borne in mind, will materially aid in their consideration.

4. A Vulgar Fraction represents a part, or parts of some thing, or number regarded as a whole.

5. It is expressed simply as an unexecuted division, with the number corresponding to the divisor below the other, and a horizontal line between them (§ 37. 3).

6. The number above the line, corresponding to the dividend, is called the *Numerator;* that below the line, corresponding to the divisor, is called the *Denominator;* and the quotient, whatever it may be, represents the *value* of the fraction.

7. The numerator and denominator together, are called *terms* of the fraction.

8. The numerator shows how many parts are denoted by the fraction.

9. The denominator shows how many of those parts make unity.

10. The denominator, therefore, gives name to the fraction. Thus, when the denominator is 2, resulting from

the division of unity into two parts, the fraction is read and called *one half;* when the denominator is 3, *thirds;* 4, *fourths;* and so on; as, $\frac{1}{2}$, one half; $\frac{2}{3}$, two thirds; $\frac{3}{4}$, three fourths.

§ 46. 1. The numerator is said to be both a dividend or number to be divided (§ 45. 6), and to denote, as the result of a division, how many parts, such as the denominator shows make unity, the fraction expresses (§ 45. 8).

2. These two objects, which it is said to serve, appear to be contradictory; and, therefore, need explanation.

3. The origin of vulgar fractions has been shown to be in a division which could not be executed—in a remainder, at the close of a division, too small to be divided by the divisor (§ 45. 1, 2). This remainder was clearly a part of the original dividend, and so was itself a dividend, but too small to contain the divisor (§ 37. 3, (3). We then wrote it at the right of the quotient figures, previously obtained, with the divisor underneath; thus indicating a division which we were not able to execute.

This expression we have regarded as a vulgar fraction, and named the numbers composing it, terms; one the numerator, the other the denominator (§ 45. 6, 7). The origin of the expression, therefore, shows the numerator to be *a dividend.*

4. Now we will show that the same expression also numbers the parts of unity denoted by the fraction (§ 45. 8).

Any fraction may be regarded as the product of a whole number into unity, divided by another number. Thus, $\frac{7}{18}=7\times\frac{1}{18}$; where 7 is the numerator, counting the parts, and equals 7×1, and 18 is the denominator, showing those parts to be eighteenths of the unit.

Now if we suppose our remainder after division to be unity, or 1, and the divisor to be 8, we shall have the expression $\frac{1}{8}$ (§ 37. 3). This 1 above the 8 is to be divided.

If we suppose this to be done, the 1 will be divided into 8 equal parts, and these, divided by the 8, would give us 1 of the 8 parts as the result; or, one eighth of 1. This, then, would be a fraction, $\frac{1}{8}$ of 1; or, as the 1 is once written in the numerator, it need not be repeated; we then abridge the expression, and write simply $\frac{1}{8}$. In this case the numerator expresses how many parts, such as the de-

nominator shows make unity, the fraction denotes; in other words, counts, or numbers, the parts (§ 45. 8).

5. The division *denoted*, and the division *executed*, then, furnish us precisely the same expression. Therefore, both assertions relative to the numerator, are consistent and true.

6. If our remainder were 7 instead of 1, and the divisor 8, 1 of the 7 divided, would, as above, give us $\frac{1}{8}$; but having 7 instead of 1, the result would be 7 times what it would be in the case of the 1, or $\frac{1}{8}\times7=\frac{7}{8}$. And so universally.

§ 47. 1. The value of the parts, as shown by the denominator, may evidently be as much varied as the numbers themselves; therefore, vulgar fractions have not, like whole numbers but a single mode of expression for the same quantity.

2. By a whole number the same quantity can be expressed but in one form; while, by vulgar fractions, the same quantity may be expressed in an infinite variety of forms. Thus, $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$, $\frac{5}{10}$, $\frac{6}{12}$, $\frac{7}{14}$, and so on, indefinitely, denote precisely the same quantity.

3. The fraction is called *Proper*, when the numerator is less than the denominator, and, therefore, cannot be divided by it; the name denoting that the dividend (the numerator) not containing the divisor (the denominator), the expression is properly a fraction, or a *proper fraction;* as, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$.

4. It is called *Improper*, when the numerator equals or exceeds the denominator, and, therefore, can be divided by it; the name denoting that the dividend (the numerator) containing the divisor (the denominator), the expression is improperly a fraction, or an *improper fraction*; as, $\frac{4}{3}$, $\frac{8}{5}$, $\frac{9}{7}$.

5. From the principle and nature of fractions, thus developed, we deduce six fundamental propositions.

§ 48. 1. PROPOSITION I. As many times as the numerator of a fraction is made larger, the denominator remaining unchanged, so many times the value of the fraction is made larger.

2. *Illustration.* $\frac{3}{7}\times6=\frac{18}{7}$. Here we multiply the numerator 3 by 6, preserving the denominator unchanged, and we obtain the result, $\frac{18}{7}$.

3. *Explanation.* When the numerator is multiplied by any number, as many more parts are denoted by it, as there are units in that number; or, as the numerator is the number to be divided—the dividend—when it is increased, so much greater is the number to be divided, and, consequently, the divisor—the denominator—remaining unchanged, so much greater is the quotient which represents the value of the fraction; hence, the value of the fraction is so many times increased.

4. One simple principle of division holds through this and the following propositions; that the larger the dividend, the divisor remaining the same, the larger the quotient; the smaller the dividend, the smaller the quotient.

5. *Examples.*

(1.) Multiply $\frac{1}{2}$ by 3. Ans. $\frac{3}{2}$.
(2.) Multiply $\frac{1}{6}$ by 9. Ans. $\frac{9}{6}$.
(3.) Multiply $\frac{3}{4}$ by 3. Ans. $\frac{9}{4}$.
(4.) Multiply $\frac{1}{12}$ by 9. Ans. $\frac{9}{12}$.
(5.) Multiply $\frac{7}{8}$ by 8. Ans. $\frac{56}{8}$.
(6.) Multiply $\frac{8}{12}$ by 6. Ans. $\frac{48}{12}$.
(7.) Multiply $\frac{7}{9}$ by 13. Ans. $\frac{91}{9}$.
(8.) Multiply $\frac{8}{10}$ by 9. Ans. $\frac{72}{10}$.
(9.) Multiply $\frac{22}{24}$ by 4. Ans. $\frac{88}{24}$.
(10.) Multiply $\frac{9}{14}$ by 5. Ans. $\frac{45}{14}$.

§ 49. 1. Proposition II. As many times as the numerator of a fraction is made smaller, the denominator remaining unchanged, so many times the value of the fraction is made smaller.

2. *Illustration.* $\frac{8}{17} \div 8 = \frac{1}{17}$. Here we divide the numerator 8 by 8, preserving the denominator unchanged, and we obtain the result, $\frac{1}{17}$.

3. *Explanation.* When the numerator is divided by any number, as many less parts are denoted by it as there are units in that number; or, as the numerator is the number to be divided—the dividend—when it is decreased, so much less is the number to be divided, and, consequently, the divisor—the denominator—remaining unchanged, so much less is the quotient, which represents the value of

the fraction; hence, the value of the fraction is so many times decreased.

4. *Examples.*

(1.) Divide $\frac{3}{8}$ by 3. Ans. $\frac{1}{8}$.
(2.) Divide $\frac{22}{50}$ by 11. Ans. $\frac{2}{50}$.
(3.) Divide $\frac{4}{7}$ by 2. Ans. $\frac{2}{7}$.
(4.) Divide $\frac{16}{42}$ by 8. Ans. $\frac{2}{42}$.
(5.) Divide $\frac{30}{43}$ by 5. Ans. $\frac{6}{43}$.
(6.) Divide $\frac{14}{17}$ by 2. Ans. $\frac{7}{17}$.
(7.) Divide $\frac{35}{40}$ by 7. Ans. $\frac{5}{40}$.
(8.) Divide $\frac{18}{22}$ by 9. Ans. $\frac{2}{22}$.
(9.) Divide $\frac{24}{25}$ by 6. Ans. $\frac{4}{25}$.
(10.) Divide $\frac{144}{200}$ by 12. Ans. $\frac{12}{200}$.

§ 50. 1. PROPOSITION. III. As many times as the denominator of a fraction is made larger, the numerator remaining unchanged, so many times the value of the fraction is made smaller.

2. *Illustration.* $\frac{3}{4}\times3=\frac{3}{12}$. Here we multiply the denominator 4 by 3, preserving the numerator unchanged, and we obtain the result, $\frac{3}{12}$.

3. *Explanation.* As many times as the denominator, which shows into how many parts unity is divided, is multiplied, into so many more parts is unity divided. And when the same thing or number is divided into more parts, so many times smaller must those parts be; therefore, so many times must the value of the fraction be diminished.

4. *Examples.*

(1.) Divide $\frac{1}{8}$ by 12. Ans. $\frac{1}{96}$.
(2.) Divide $\frac{14}{15}$ by 9. Ans. $\frac{14}{135}$.
(3.) Divide $\frac{2}{5}$ by 18. Ans. $\frac{2}{90}$.
(4.) Divide $\frac{11}{19}$ by 5. Ans. $\frac{11}{95}$.
(5.) Divide $\frac{4}{21}$ by 7. Ans. $\frac{4}{147}$.
(6.) Divide $\frac{2}{10}$ by 19. Ans. $\frac{2}{190}$.
(7.) Divide $\frac{25}{37}$ by 4. Ans. $\frac{25}{148}$.
(8.) Divide $\frac{18}{23}$ by 6. Ans. $\frac{18}{138}$.
(9.) Divide $\frac{2}{8}$ by 33. Ans. $\frac{2}{264}$.

(10.) Divide $\frac{15}{16}$ by 8. Ans. $\frac{15}{128}$.
(11.) Divide $\frac{2}{5}$ by 19. Ans. $\frac{2}{95}$.
(12.) Divide $\frac{17}{32}$ by 3. Ans. $\frac{17}{96}$.
(13.) Divide $\frac{7}{8}$ by 20. Ans. $\frac{7}{160}$.

§ 51. 1. Proposition IV. As many times as the denominator of a fraction is made smaller, the numerator remaining unchanged, so many times the value of the fraction is made larger.

2. *Illustration.* $\frac{3}{16} \div 16 = \frac{3}{1} = 3$. Here we divide the denominator 16 by 16, preserving the numerator unchanged, and we obtain the result, $\frac{3}{1}=3$.

3. *Explanation.* The smaller the number of parts into which unity is divided, the larger must be those parts. Taking then the same number of the parts, increased in size, we take so much more as the parts are made larger, or as they are diminished in number.

4. *Examples.*

(1.) Multiply $\frac{2}{21}$ by 7. Ans. $\frac{2}{3}$.
(2.) Multiply $\frac{9}{54}$ by 6. Ans. $\frac{9}{9}$.
(3.) Multiply $\frac{11}{128}$ by 16. Ans. $\frac{11}{8}$.
(4.) Multiply $\frac{3}{20}$ by 4. Ans. $\frac{3}{5}$.
(5.) Multiply $\frac{8}{9}$ by 3. Ans. $\frac{8}{3}$.
(6.) Multiply $\frac{4}{84}$ by 7. Ans. $\frac{4}{12}$.
(7.) Multiply $\frac{19}{288}$ by 24. Ans. $\frac{19}{12}$.
(8.) Multiply $\frac{14}{72}$ by 3. Ans. $\frac{14}{24}$.
(9.) Multiply $\frac{18}{27}$ by 9. Ans. $\frac{18}{3}$.
(10.) Multiply $\frac{7}{56}$ by 7. Ans. $\frac{7}{8}$.
(11.) Multiply $\frac{14}{28}$ by 4. Ans. $\frac{14}{7}$.
(12.) Multiply $\frac{13}{15}$ by 5. Ans. $\frac{13}{3}$.

§ 52. 1. Proposition V. Both terms of a fraction being multiplied by the same number, the value of the fraction remains unchanged.

2. *Illustration.* $\frac{6\times3}{9\times3}=\frac{18}{27}$.

Here we multiply both terms of the fraction $\frac{6}{9}$ by 3, and we obtain the result, $\frac{18}{27}$.

3. *Explanation.* This is a combination of Propositions I. and III. The multiplication of the numerator, agreeably to the former, increases the value of the fraction, while the multiplication of the denominator, agreeably to the latter, decreases its value; and if both are multiplied by the same number, the value of the fraction is increased as much by the multiplication of the numerator as it is diminished by the multiplication of the denominator. The multiplication of both terms therefore, by the same number, makes an exact compensation, and the value of the fraction remains unchanged.

4. *Examples.*

(1.) Multiply both terms of $\frac{7}{12}$ by 3. Ans. $\frac{21}{36}$.
(2.) Multiply both terms of $\frac{8}{9}$ by 5. Ans. $\frac{40}{45}$.
(3.) Multiply both terms of $\frac{6}{7}$ by 13. Ans. $\frac{78}{91}$.
(4.) Multiply both terms of $\frac{19}{22}$ by 4. Ans. $\frac{76}{88}$.
(5.) Multiply both terms of $\frac{17}{57}$ by 6. Ans. $\frac{102}{342}$.
(6.) Multiply both terms of $\frac{9}{10}$ by 21. Ans. $\frac{189}{210}$.
(7.) Multiply both terms of $\frac{5}{8}$ by 9. Ans. $\frac{45}{72}$.
(8.) Multiply both terms of $\frac{73}{89}$ by 5. Ans. $\frac{365}{445}$.
(9.) Multiply both terms of $\frac{3}{7}$ by 12. Ans. $\frac{36}{84}$.
(10.) Multiply both terms of $\frac{14}{53}$ by 2. Ans. $\frac{28}{106}$.

§ 53. 1. Proposition VI. Both terms of a fraction being divided by the same number, the value of the fraction remains unchanged.

2. *Illustration.* $\frac{6 \div 3}{9 \div 3} = \frac{2}{3}$.

Here we divide both terms of the fraction $\frac{6}{9}$ by 3, and we obtain the result, $\frac{2}{3}$.

3. *Explanation.* This is a combination of Propositions II. and IV. The division of the numerator, agreeably to the former, diminishes the value of the fraction; while the division of the denominator, agreeably to the latter, increases it, and if it be by the same number, it is evident the operations will just compensate, the one for the other, and so the value of the fraction will remain unchanged.

4. *Examples.*

(1.) Divide both terms of $\frac{12}{28}$ by 4. Ans. $\frac{3}{7}$.
(2.) Divide both terms of $\frac{48}{64}$ by 8. Ans. $\frac{6}{8}$.
(3.) Divide both terms of $\frac{84}{108}$ by 12. Ans. $\frac{7}{9}$.
(4.) Divide both terms of $\frac{76}{80}$ by 4. Ans. $\frac{19}{20}$.
(5.) Divide both terms of $\frac{144}{288}$ by 24. Ans. $\frac{6}{12}$.
(6.) Divide both terms of $\frac{49}{56}$ by 7. Ans. $\frac{7}{8}$.
(7.) Divide both terms of $\frac{81}{117}$ by 9. Ans. $\frac{9}{13}$.
(8.) Divide both terms of $\frac{54}{84}$ by 6. Ans. $\frac{9}{14}$.
(9.) Divide both terms of $\frac{95}{115}$ by 5. Ans. $\frac{19}{23}$.

5. The first four propositions above, solve directly all multiplication or division of fractions by whole numbers, in a double manner; for we have evidently, every time the choice between two operations, each of which may, according to the case, present a preference in a given instance.

(1.) In multiplying the numerator, we multiply the fraction.

(2.) In dividing the denominator, we multiply the fraction.

(3.) In dividing the numerator, we divide the fraction.

(4.) In multiplying the denominator, we divide the fraction.

6. The last two propositions furnish the means of changing fractions from one name or denomination to certain others, without altering their value.

§ 54. 1. What has now been developed prepares us for some primary operations in fractions, with which, at this stage, it is proper to become familiar.

2. It is often necessary to express a whole number fractionally, especially in combining whole numbers and fractions. The fraction thus obtained, it is evident, will be improper (§ 47. 4).

3. Every whole number may be expressed as a fraction, with unity for a denominator. For, by placing this denominator under any number, we do not vary its value, as the quotient obtained by dividing any number by unity, will always equal the number divided. $3=\frac{3}{1}=3$.

4. Every whole number may also have any other number for a denominator. Hence we have,

§ 55. 1. CASE I. *To change a whole number to an equivalent fraction of a specified denominator.*

RULE. Multiply the whole number by the given denominator, and, under the product, write that denominator for the fraction required.

2. *Illustration.* Change 3 to fourths.

$$\frac{3\times4}{4}=\frac{12}{4}.$$

(1.) Here we first indicate the operation to be performed; that 3, the given number, is to be multiplied by 4, the given denominator, and that the denominator to the product is to be 4, by writing the 4 underneath.

(2.) We then execute the multiplication, and obtain 12 for the numerator. Under this we write the 4, the given denominator, and we have the result, $\frac{12}{4}$.

3. *Explanation.* When the whole number is multiplied by the specified denominator, the result is a whole number, whose value is so many times greater than that required, as the denominator indicates. This result is changed to a fraction by writing the denominator under it; and the value of the expression is thus diminished as many times as it was increased by the multiplication. We have then for the whole number given, an equivalent fraction with the denominator specified. This operation is simply that of multiplying and dividing the same expression by the same number; which two operations (§ 52. 3) exactly compensating each other, the value of the given expression remains unchanged.

4. The object of an operation of this kind is merely to change the form of an expression, to answer certain ends in combining fractions, or whole numbers and fractions.

5. *Examples.*

(1.) Change 5 to fifths. Ans. $\frac{25}{5}$.
(2.) Change 4 to sevenths. Ans. $\frac{28}{7}$.
(3.) Change 8 to tenths. Ans. $\frac{80}{10}$.
(4.) Change 9 to eighths. Ans. $\frac{72}{8}$.
(5.) Change 12 to twenty-fifths. Ans. $\frac{300}{25}$.

(6.) Change 17 to thirty-thirds. Ans. $\frac{561}{33}$.
(7.) Change 19 to thirty-fifths. Ans. $\frac{665}{35}$.
(8.) Change 24 to ninths. Ans. $\frac{216}{9}$.
(9.) Change 30 to thirteenths. Ans. $\frac{390}{13}$.
(10.) Change 43 to sixths. Ans. $\frac{258}{6}$.

§ 56. It is often necessary, when there is a fraction annexed to a whole number making a mixed expression, or as it is usually called *a mixed number*, to change the expression to a fraction. Hence we have,

§ 57. 1. CASE II. *To change a mixed number to an equivalent fraction.*

RULE. Multiply the whole number by the denominator of the fraction; to the product add the numerator; and, under the sum, write the denominator of the fraction, for the fraction required.

2. *Illustration*, Change $4\frac{1}{3}$ to a fraction,

$$\frac{4\times3+1}{3}=\frac{13}{3},$$

(1.) Here, as in the last case, we first indicate the operation to be performed; that 4, the whole number, is to be multiplied by 3, the denominator of the fraction, and 1, the numerator of the fraction, to be added to the product; and that the denominator to the sum is to be 3, the denominator to the fraction, by writing the 3 underneath.

(2.) We then execute the multiplication of the 3 and 4, and addition to the product of 1, and obtain 13. Under this sum, we write the 3, the denominator of the fraction, and we have the result, $\frac{13}{3}$.

3. *Explanation.* This operation first changes the whole number to an equivalent fraction, with the denominator to the annexed fraction for its denominator. The expression is then as much less than the mixed number given, as the parts denoted by the fraction annexed; and to make it equivalent, those parts, expressed by the numerator of the fraction (§ 45. 8), are joined to it.

4. The object of this operation is the same with the last

(§ 55. 4)—to give to the whole number a specified denominator—that of the fraction annexed.

The fraction thus obtained, it is obvious is an improper fraction.

5. *Examples.*

(1.) Change $5\frac{2}{3}$ to a fraction. Ans. $\frac{17}{3}$.
(2.) Change $7\frac{4}{5}$ to a fraction. Ans. $\frac{39}{5}$.
(3.) Change $9\frac{8}{12}$ to a fraction. Ans. $\frac{116}{12}$.
(4.) Change $12\frac{2}{14}$ to a fraction. Ans. $\frac{170}{14}$.
(5.) Change $21\frac{12}{18}$ to a fraction. Ans. $\frac{390}{18}$.
(6.) Change $33\frac{19}{20}$ to a fraction. Ans. $\frac{679}{20}$.
(7.) Change $42\frac{3}{15}$ to a fraction. Ans. $\frac{633}{15}$.
(8.) Change $38\frac{9}{11}$ to a fraction. Ans. $\frac{427}{11}$.
(9.) Change $10\frac{8}{9}$ to a fraction. Ans. $\frac{98}{9}$.

§ 58. It is often necessary to reverse the last step. Hence we have,

§ 59. 1. CASE III. *To change an improper fraction to an equivalent whole or mixed number.*

RULE. Divide the numerator by the denominator, and the quotient will be the whole or mixed number required.

2. *Illustration,* Change $\frac{16}{5}$ to a mixed number.

$$\tfrac{16}{5}=3\tfrac{1}{5}.$$

Here we simply execute the division indicated, and we obtain the result, $3\frac{1}{5}$.

3. *Explanation.* This operation is just the reverse of that in Case II,: it is merely the execution of an indicated division (§ 45. 6),

4. *Examples,*

(1.) Change $\frac{22}{8}$ to a mixed number. Ans. $2\frac{6}{8}$.
(2.) Change $\frac{19}{6}$ to a mixed number. Ans. $3\frac{1}{6}$.
(3.) Change $\frac{25}{9}$ to a mixed number. Ans. $2\frac{7}{9}$.
(4.) Change $\frac{478}{10}$ to a mixed number. Ans. $47\frac{8}{10}$.
(5.) Change $\frac{589}{12}$ to a mixed number. Ans. $49\frac{1}{12}$.

(6.) Change $\frac{390}{18}$ to a mixed number. Ans. $21\frac{12}{18}$.
(7.) Change $\frac{633}{15}$ to a mixed number. Ans. $42\frac{3}{15}$.
(8.) Change $\frac{7341}{213}$ to a mixed number. Ans. $34\frac{99}{213}$.
(9.) Change $\frac{1417}{60}$ to a mixed number. Ans. $23\frac{37}{60}$.
(10.) Change $\frac{8763}{12}$ to a mixed number. Ans. $730\frac{3}{12}$.

§ 60. The terms of a fraction are often susceptible of being divided by the same number, and so of being diminished in the same degree;—or, as the same thing is sometimes called, of being reduced proportionally, or canceled; for by the operation the same factors in each term are suppressed or canceled.

The terms then are lower, though the fraction be expressive of the same quantity (§ 53. 1). Hence we have,

§ 61. 1. Case IV. *To change a fraction to its equivalent lowest expression.*

Rule. I. Divide both terms of the fraction by any number that will divide them without a remainder, and those quotients again in the same manner, and so on, till no number greater than unity will divide them.

II. The last quotients will be the lowest expression required.

2. *Illustration.* Change $\frac{144}{288}$ to its lowest expression.

$$\frac{144}{288}\div 12=\frac{12}{24}\div 12=\frac{1}{2} \text{ or, } \frac{144}{288}\div 144=\frac{1}{2}.$$

(1.) Here we divide first by 12, perceiving that it will divide both terms of the fraction, and the quotients thus obtained by 12 again, for the same reason, and we have the result, $\frac{1}{2}$.

(2.) Or, perceiving that 144 will divide both terms, and knowing this to be the largest number that will do it, we divide by it, and thus, by a single division, have the result, $\frac{1}{2}$.

(3.) Or, better and shorter, we may keep the divisor or divisors in mind, without writing them, and at once divide, writing down only the quotients.

Each number has, in this case, a line drawn across it, as soon as the division has been executed, and the quotient is then written. This is properly *canceling*.

This mode is almost uniformly preferable to either of the others; therefore the pupil should be much exercised in it, that he may perform it with ease, facility and exactness.

Perhaps, for this operation, it will generally be found most convenient to write the terms with a perpendicular instead of a horizontal line between them; the numerator, as the dividend, on the right; the denominator, as the divisor, on the left. Then the quotients arising from the division of the respective terms by the same number, may be written either above, below, at the right or left of those terms, as may be most convenient.

3. *Explanation.* The explanation to Proposition VI., is the explanation for this operation.

4. *Examples.*

(1.) Change $\frac{21}{35}$ to its lowest expression. Ans. $\frac{3}{5}$.
(2.) Change $\frac{128}{160}$ to its lowest expression. Ans. $\frac{4}{5}$.
(3.) Change $\frac{165}{275}$ to its lowest expression. Ans. $\frac{3}{5}$.
(4.) Change $\frac{99}{297}$ to its lowest expression. Ans. $\frac{1}{3}$.
(5.) Change $\frac{140}{160}$ to its lowest expression. Ans. $\frac{7}{8}$.
(6.) Change $\frac{450}{900}$ to its lowest expression. Ans. $\frac{1}{2}$.
(7.) Change $\frac{275}{440}$ to its lowest expression. Ans. $\frac{5}{8}$.
(8.) Change $\frac{315}{405}$ to its lowest expression. Ans. $\frac{7}{9}$.
(9.) Change $\frac{1644}{2192}$ to its lowest expression. Ans. $\frac{3}{4}$.
(10.) Change $\frac{468}{1184}$ to its lowest expression. Ans. $\frac{117}{296}$.
(11.) Change $\frac{1429}{2858}$ to its lowest expression. Ans. $\frac{1}{2}$.

§ 62. Another mode of changing a fraction to its lowest expression is given, in addition to this, in most treatises on Arithmetic; that of finding what is called the greatest common measure of the two terms, and then dividing both terms by it. That, however, is in most cases, a longer process than this, and its principle much less simple. It belongs, more properly, to Algebra.

We prefer, in Arithmetic, the mode here presented, because it is clear and simple, and is so easily made intelligible to the pupil.

§ 63. Parts of fractions, or fractions of fractions, usually called *compound fractions*, often occur, which it is neces-

sary to change to a single expression. Hence we have,

§ 64. 1. CASE V. *To change a compound fraction to an equivalent single one.*

RULE. Multiply all the numerators together, and all the denominators together, and the products will be respectively the numerator and denominator of the required fraction.

2. *Illustration.* Change the compound fraction, $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$ to a single one.

$$\begin{array}{r|l} 2 & 1 \\ 4 & 3 \\ 5 & 4 \\ 6 & 5 \\ \hline 240 & 60=\frac{60}{240}\div 60=\frac{1}{4}. \end{array}$$

(1.) Here we draw a perpendicular line, corresponding to the curved line for the divisor in division, and write all the denominators, which are divisors (§ 45. 6), at the left of it, and all the numerators, which are dividends, (§ 45. 6), at its right, and, under each other, that they may more conveniently be multiplied together; and draw a horizontal line below.

(2.) We then multiply all the denominators together, in mind, beginning with the 6; saying, 6 times 5 are 30; 4 times 30 are 120; 2 times, or twice 120 are 240; which is the product of all the denominators. This we write below.

(3.) We then multiply, in the same way, all the numerators together, and we have their product 60, which we write under the numerators.

(4.) Then the expression is read $\frac{60}{240}$; or we write the sign of equality, and express the result fractionally.

3. Or, better and shorter, we write the numbers as above, and then cancel, as far as possible, and multiply the uncanceled numbers which remain, and the quotients together (§ 60.). We thus, at once, have a true result, and that in the lowest expression.

4. By this mode, the operation on the fraction above would be,

$$\begin{array}{r|l} 2 & 1 \\ 1\ \cancel{4} & \cancel{3}\ 1 \\ 1\ \cancel{5} & \cancel{4}\ 1 \\ 2\ \cancel{6} & \cancel{5}\ 1 \\ \hline 4 & 1=\frac{1}{4}. \end{array}$$

(1.) Here we divide the numbers on each side of the perpendicular line, by such numbers as will divide them without remainder, and write down the quotients.

(2.) We first divide by 4. This divides 4 on the left and the same on the right. These two figures, therefore, are scratched, to show that they have been divided or canceled, and the quotient 1, in each case is written down.

(3.) We then divide in the same way by 5, and scratch the figures divided as before, and write down the quotients.

(4.) Then we divide by 3 in the same way, scratch the figures divided, and write down the quotients as before.

(5.) As the division can be continued no further, we then draw the line underneath, multiply together the unscratched figures, and write the products below the line. These then furnish the result.

5. *Explanation.* A compound fraction is simply a part of a part, and that it may be represented by a single one, is evident from the fact, that every part of a part or parts, must be equal to some part of a whole.

6. Now, if the part of a part required be but 1, it is evident (§ 45. 9) that this is obtained by multiplying the denominator of the part which is the divisor, by the denominator of the part to be divided (§ 50. 1).

7. Hence it will be obvious, that if the part of a part required be more than 1, that number is only to be multiplied into the numerator of the other to obtain the result (§ 48. 1).

8. Let the compound fraction to be changed be, $\frac{3}{4}$ of $\frac{4}{5}$. Then $\frac{1}{4}$ of $\frac{4}{5}=\frac{4}{5}\div4=\frac{4}{20}$ (§ 50. 1). And if the fraction be $\frac{3}{4}$ of $\frac{4}{5}$, then $\frac{4}{5}\div4$, or, $\frac{1}{5}=\frac{4}{20}$; and $\frac{4}{5}\div\frac{3}{4}=\frac{4}{20}\times3=\frac{12}{20}$ (§48. 1) $=\frac{3}{5}$.

9. The same reasoning will apply if the compound fraction consist of more expressions than two. For, it is evident, that the first two may be changed to one; and then that and the next be changed to one, and so on.

6. *Examples.*

(1.) Change $\frac{3}{4}$ of $\frac{2}{3}$ to a single fraction. Ans. $\frac{1}{2}$.

(2.) Change $\frac{4}{5}$ of $\frac{2}{3}$ of $\frac{7}{8}$ of $\frac{3}{4}$ to a single fraction. Ans. $\frac{7}{20}$.

(3.) Change $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ to a single fraction. Ans. $\frac{1}{5}$.

(4.) Change $\frac{5}{7}$ of $\frac{7}{9}$ of $\frac{3}{5}$ of $\frac{3}{4}$ to a single fraction. Ans. $\frac{1}{4}$.

(5.) Change $\frac{14}{17}$ of $\frac{18}{31}$ of $\frac{17}{36}$ of $\frac{5}{8}$ to a single fraction. Ans. $\frac{35}{248}$.

(6.) Change $\frac{9}{10}$ of $\frac{8}{11}$ of $\frac{4}{6}$ of $\frac{22}{24}$ to a single fraction. Ans. $\frac{2}{5}$.

(7.) Change $\frac{8}{16}$ of $\frac{2}{5}$ of $\frac{5}{9}$ of $\frac{7}{8}$ to a single fraction. Ans. $\frac{7}{72}$.

(8.) Change $\frac{2}{3}$ of $\frac{18}{20}$ of $\frac{11}{12}$ of $\frac{33}{36}$ to a single fraction. Ans. $\frac{121}{240}$.

§ 65. 1. The opposite of a compound fraction, or that to which the compound fraction is changed, is usually called a *simple fraction.* But, as the object of the operation is to have a single expression for many, we prefer the name *single fraction.*

2. If, in operations of this kind, mixed numbers occur, they should first be changed to improper fractions (§ 57. 1).

§ 66. To combine fractions by addition, or separate them by subtraction, and for other operations, it is often desired to change them to the same name or denomination, which is done by changing them to the same, or, as it is usually called, to a *common denominator.* Hence we have,

§ 67. 1. CASE VI. *To change fractions of different denominators to equivalent fractions with a common denominator.*

RULE. Multiply the numerator and denominator of each fraction by all the denominators except its own.

2. *Illustration.* Change $\frac{3}{5}$, $\frac{5}{6}$, $\frac{7}{8}$, and $\frac{8}{9}$, to equivalent fractions having a common denominator.

$$\frac{3}{5} \quad \frac{5}{6} \quad \frac{7}{8} \quad \frac{8}{9}$$

$$\begin{array}{r|l|l|l|l}
5 & 3 & 5 & 7 & 8 \\
6 & 6 & 5 & 5 & 5 \\
8 & 8 & 8 & 6 & 6 \\
9 & 9 & 9 & 9 & 8 \\
\hline
2160 & 1296 & 1800 & 2520 & 1920
\end{array} = \frac{1296+1800+2520+1920}{2160}$$

$$\overline{\quad 2160 \quad}$$

(1.) Here we draw a perpendicular line under each fraction, and write the denominators under each other at the left of the left line.

(2.) We then write the numerators, one at the right of each line, and under each all the denominators except its own.

The denominators are thus written but once, because the denominator to be obtained is to be the same for all the numerators.

(3.) Then we multiply all the denominators together, and we obtain 2160, as the common denominator. We then multiply together the numbers at the right of each line, for the numerators of the respective fractions; and we obtain 1296, 1800, 2520, and 1920, as the numerators required (§ 44. 1).

(4.) These, then, may be read as they stand, mentioning the denominator with the last; or the sign of equality may be written at the right, and they be written at the right of it, with the denominator under them; or, shorter, the denominator may be written under them, as they stand.

3. *Explanation.* It is evident that in this operation, both terms of the fractions being multiplied by precisely the same numbers, the value of the fractions remains unchanged (§ 52. 1).

4. *Examples.*

(1.) Change $\frac{1}{2}$, $\frac{2}{3}$, $\frac{7}{8}$, and $\frac{4}{5}$ to equivalent fractions having a common denominator. Ans. $\frac{120}{240}$, $\frac{160}{240}$, $\frac{210}{240}$, $\frac{192}{240}$.

(2.) Change $\frac{1}{3}$, $\frac{3}{5}$, and $\frac{1}{4}$ to equivalent fractions having a common denominator. Ans. $\frac{20}{60}$, $\frac{36}{60}$, $\frac{15}{60}$.

(3.) Change $\frac{2}{3}$, $\frac{1}{2}$, and $\frac{3}{4}$ to equivalent fractions having a common denominator. Ans. $\frac{16}{24}$, $\frac{12}{24}$, $\frac{18}{24}$.

(4.) Change $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{8}$ to equivalent fractions having a common denominator.

Ans. $\frac{576}{1152}$, $\frac{768}{1152}$, $\frac{864}{1152}$, $\frac{960}{1152}$, $\frac{1008}{1152}$.

(5.) Change $\frac{1}{2}$, $\frac{10}{18}$, $\frac{3}{4}$, and $\frac{3}{13}$ to equivalent fractions having a common denominator. Ans. $\frac{936}{1872}$, $\frac{1040}{1872}$, $\frac{1404}{1872}$, $\frac{432}{1872}$.

(6.) Change $\frac{11}{15}$, $\frac{3}{8}$, $\frac{7}{12}$, and $\frac{5}{8}$ to equivalent fractions having a common denominator.

Ans. $\frac{8448}{11520}$, $\frac{4320}{11520}$, $\frac{6720}{11520}$, $\frac{7200}{11520}$.

(7.) Change $\frac{1}{4}$, $\frac{2}{5}$, and $\frac{5}{8}$ to equivalent fractions having a common denominator. Ans. $\frac{40}{160}$, $\frac{64}{160}$, $\frac{100}{160}$.

§ 68. To shorten operations, it is often desirable to obtain a common denominator and the lowest expressions for the fractions in the same operation. Hence we have,

§ 69. 1. CASE VII. *To change fractions of different denominators to equivalent fractions with a least common denominator.*

RULE. I. Write the terms of the fractions as in Case VI.

II. Cancel the figures that are common to each column, or the factors that are common in each.

III. In each column, multiply together the undivided numbers and the quotients thus obtained, and the product of the column at the left will be the least common denominator, and the products of the others will be the respective numerators for the fractions required.

2. *Illustration.* Change $\frac{3}{5}$, $\frac{5}{6}$, $\frac{7}{8}$, and $\frac{8}{9}$ to equivalent fractions, having a least common denominator.

$$\frac{3}{5} \quad \frac{5}{6} \quad \frac{7}{8} \quad \frac{8}{9}$$

5	3	$\cancel{5}$	7	8
$3\cancel{6}$	$\cancel{6}3$	5	5	5
8	8	$\cancel{8}4$	$\cancel{6}\cancel{3}$	$\cancel{6}\cancel{3}$
$3\cancel{9}$	$\cancel{9}3$	$\cancel{9}3$	9	8
360	216	300	315	320

360

(1.) Here we write the terms of the fractions as in Case VI.

(2.) Then we cancel by dividing the column of denominators and those of the numerators, first by 2, and then by 3.

(3.) We then find that they cannot all be divided farther by any one number; we conclude, therefore, that they are in their lowest expression.

(4.) Then multiplying the numbers remaining undivided, and the quotients obtained by the divisions together, we have for the least common denominator 360, and for the numerators, 216, 300, 315, 320. Under these we write the denominator, and the operation is completed.

3. *Explanation.* This operation involves only the principle of Proposition VI., and Case IV. It is simply dividing both terms of the fraction by the same number or numbers, or canceling. The explanations referred to are sufficient for it.

4. Fractions of different denominators are usually changed to equivalent fractions with a least common denominator, by a process very different from that here adopted. There is first obtained the least common multiple of all the denominators; that is, the least number that can be divided by them all without a remainder, and from this the fractions are obtained.

This process, however, is much longer, and less intelligible than that here adopted.

5. *Examples.*

(1.) Change $\frac{1}{2}$, $\frac{3}{8}$, and $\frac{5}{6}$ to equivalent fractions having a least common denominator. Ans. $\frac{12}{24}$, $\frac{9}{24}$, $\frac{20}{24}$.

(2.) Change $\frac{1}{6}$ and $\frac{1}{9}$ to equivalent fractions having a least common denominator. Ans. $\frac{3}{18}$, $\frac{2}{18}$.

(3.) Change $\frac{3}{4}$, $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{1}{6}$ to equivalent fractions having a least common denominator. Ans. $\frac{9}{12}$, $\frac{6}{12}$, $\frac{8}{12}$, $\frac{2}{12}$.

(4.) Change $\frac{1}{3}$, $\frac{3}{4}$, and $\frac{7}{8}$ to equivalent fractions having a least common denominator. Ans. $\frac{8}{24}$, $\frac{18}{24}$, $\frac{21}{24}$.

(5.) Change $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$ to equivalent fractions having a least common denominator. Ans. $\frac{30}{60}$, $\frac{40}{60}$, $\frac{45}{60}$, $\frac{48}{60}$.

(6.) Change $\frac{3}{4}$, $\frac{5}{8}$, $\frac{2}{7}$, and $\frac{8}{21}$ to equivalent fractions having a least common denominator.

Ans. $\frac{126}{168}$, $\frac{105}{168}$, $\frac{48}{168}$, $\frac{64}{168}$.

§ 70. It often occurs that a fraction has a fraction for a denominator. Such an expression is called a *complex fraction*, and it may be necessary to change it to a single one. Hence we have,

§ 71. 1. CASE VIII. *To change a complex fraction to an equivalent single one.*

RULE. Multiply both terms of each fraction by the denominator of the other. Cancel the denominators thus obtained in each, and the fraction resulting from the numerators will be the single fraction required.

2. *Illustration.* Change $\frac{\frac{5}{6}}{\frac{3}{4}}$ to an equivalent single fraction.

$$\frac{\frac{5}{6}\times 4}{\frac{3}{4}\times 6}=\frac{\frac{20}{24}}{\frac{18}{24}}=\frac{20}{18}.$$

3. *Explanation.* It is obvious that the principle of the first part of this operation is simply that of Case VI., changing fractions to a common denominator; for the numerator and denominator of each fraction are multiplied by every denominator except its own; and that that of the second is sustained by Proposition VI., where it is shown that both the numerator and denominator of a fraction being divided by the same number, the value of the fraction remains the same.

4. When the pupil has become familiar with this operation, he may shorten it essentially by neglecting to multiply the denominators, since their products are always cancelled, and multiply only the numerators; as, $\frac{\frac{5}{6}\times 4}{\frac{3}{4}\times 6}=\frac{20}{18}$. The required single fraction is thus at once obtained.

5. *Examples.*

(1.) Change $\frac{\frac{1}{2}}{\frac{2}{3}}$ to a single fraction. Ans. $\frac{3}{4}$.

(2.) Change $\frac{\frac{5}{6}}{\frac{7}{8}}$ to a single fraction. Ans. $\frac{40}{42}$.

(3.) Change $\frac{\frac{4}{5}}{\frac{9}{10}}$ to a single fraction. Ans. $\frac{40}{45}$.

(4.) Change $\frac{\frac{3}{4}}{\frac{11}{12}}$ to a single fraction. Ans. $\frac{36}{44}$.

(5.) Change $\frac{\frac{1}{12}}{\frac{5}{9}}$ to a single fraction. Ans. $\frac{9}{60}$.

(6.) Change $\frac{\frac{3}{8}}{\frac{4}{7}}$ to a single fraction. Ans. $\frac{21}{32}$.

(7.) Change $\frac{\frac{6}{7}}{\frac{9}{11}}$ to a single fraction. Ans. $\frac{66}{63}$.

(8.) Change $\frac{\frac{6}{7}}{\frac{1}{3}}$ to a single fraction. Ans. $\frac{18}{7}$.

(9.) Change $\frac{\frac{12}{13}}{\frac{1}{6}}$ to a single fraction. Ans. $\frac{72}{13}$.

(10.) Change $\frac{\frac{1}{3}}{\frac{3}{4}}$ to a single fraction. Ans. $\frac{4}{9}$.

§ 72. The operations of Addition, Multiplication, Subtraction and Division, are performed on fractions as on whole numbers.

The previous steps having prepared us for these operations, we now present them.

§ 73. 1. For the operation of Addition it is evidently necessary that the fractions represent the same divisions of unity; in other words, that they have the same denominator, otherwise the quantities they represent would be unlike in kind, and so we should bring together parts not the same. The impropriety of this is obvious.

2. The parts being made the same, we add the numerators as we add whole numbers.

3. As by the process we have adopted, it is shorter and easier to obtain the least common denominator, than to obtain only a common denominator, and as in the result this is usually desired, we prefer for this operation at once to change the fractions to their least common denominator (§69. 1). Hence we have,

§ 74. 1. Case IX. *To add Vulgar Fractions.*

Rule. Change the fractions to their least common denominator, add the numerators, and, under the sum, write the denominator.

2. *Illustration.* Add $\frac{3}{5}+\frac{5}{6}+\frac{7}{8}+\frac{8}{9}$ together.

$$\frac{3}{5}+\frac{5}{6}+\frac{7}{8}+\frac{8}{9}$$

5	3	5	7	8
3$\cancel{6}$	$\cancel{6}$3	5	5	8
8	8	$\cancel{8}$4	$\cancel{6}\cancel{3}$	5
3$\cancel{9}$	$\cancel{9}$3	$\cancel{9}$3	$\cancel{9}$	$\cancel{6}\cancel{3}$
360	216	300	315	320$=\frac{1151}{360}$.

(1.) Here we write the fractions as in Cases VI. and VII.; then by canceling change them to their least common denominator; then add the numerators, and under the sum 1151 write the least common denominator 360, and we have the required sum of the fractions.

(2.) All the steps in this operation having been made clear by previous illustrations and explanations, no explanation here is required.

3. *Examples.*

(1.) Add $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$ together. Ans. $1\frac{11}{12}$.
(2.) Add $\frac{3}{5}$, $\frac{5}{6}$, and $\frac{2}{3}$ together. Ans. $2\frac{1}{10}$.
(3.) Add $\frac{7}{12}$, $\frac{11}{18}$, and $\frac{14}{36}$ together. Ans. $1\frac{7}{12}$.
(4.) Add $\frac{3}{7}$, $9\frac{1}{5}$, and $\frac{2}{3}$ of $\frac{1}{2}$ together. Ans. $9\frac{101}{105}$.
(5.) Add $\frac{4}{5}$, $\frac{5}{6}$ of $\frac{3}{8}$ of $\frac{1}{4}$, and $8\frac{3}{14}$ together. Ans. $9\frac{207}{2240}$.
(6.) Add $\frac{1}{4}$, $\frac{3}{7}$, and $\frac{5}{6}$ together. Ans. $1\frac{43}{84}$.
(7.) Add $\frac{1}{5}$, $4\frac{1}{2}$, and $\frac{2}{5}$ together. Ans. $5\frac{1}{10}$.
(8.) Add $\frac{5}{8}$, $7\frac{1}{2}$, and $\frac{1}{3}$ of $\frac{3}{4}$ together. Ans. $8\frac{3}{8}$.
(9.) Add $\frac{7}{8}$ of $\frac{9}{10}$, $\frac{4}{7}$ of $\frac{1}{2}$, and $7\frac{1}{2}$ together. Ans. $8\frac{321}{560}$.
(10.) Add $\frac{2}{7}$, and $\frac{2}{3}$ of $\frac{3}{4}$ together. Ans. $\frac{11}{14}$.
(11.) Add $\frac{3}{7}$, $\frac{4}{2}$, and $\frac{3}{5}$ together. Ans. $3\frac{1}{35}$.
(12.) Add $\frac{9}{11}$, $\frac{11}{14}$ and $\frac{5}{6}$ together. Ans. $2\frac{101}{231}$.
(13.) Add $\frac{4}{8}$, $\frac{3}{4}$ of $\frac{7}{10}$, and $\frac{8}{9}$ of $\frac{2}{3}$ together. Ans. $1\frac{667}{1080}$.
(14.) Add $\frac{3}{6}$, $\frac{2}{3}$, $\frac{4}{5}$, and $\frac{8}{12}$ together. Ans. $2\frac{19}{30}$.
(15.) Add $\frac{4}{9}$, $\frac{3}{4}$, $\frac{9}{10}$, and $\frac{3}{7}$ together. Ans. $2\frac{659}{1260}$.

(16.) Add $18\frac{3}{4}$, $5\frac{1}{2}$, $7\frac{5}{16}$, $8\frac{2}{3}$, and $9\frac{7}{8}$ together. Ans. $50\frac{5}{48}$.

(17.) Add $789\frac{4}{7}$ years, $817\frac{2}{9}$ years, $316\frac{5}{6}$ years, and $216\frac{1}{2}$ years together. Ans. $2140\frac{8}{63}$.

(18.) Add $819\frac{3}{4}$ barrels, $\frac{1}{5}$ of $\frac{5}{8}$ barrels, $409\frac{3}{8}$ barrels, and $\frac{1}{2}$ of $5\frac{1}{2}$ barrels together. Ans. 1232.

(19.) Add $\dfrac{\frac{3}{4}}{8}$, $\dfrac{6}{\frac{3}{7}}$, and $\dfrac{\frac{4}{5}}{\frac{8}{15}}$ together. Ans. $15\frac{19}{32}$.

§ 75. Fractions are multiplied with greater ease and more facility than they are added. For, whatever may be the denominators, the operation is at once performed without at all changing the fractions as is done for Addition. The manner of performing this operation is obvious from Propositions I. and III., and § 64. 1. Hence we have,

§ 76. 1. CASE X. *To multiply a fraction by a whole number; or, a whole number by a fraction.*

RULE. Multiply the whole number by the numerator of the fraction, and divide the product by the denominator.

2. *Illustration.* Multiply 16 by $\frac{2}{3}$.

$$\begin{array}{r} 16\times2 \\ \hline 3)32 \\ \hline 10\frac{2}{3} \end{array}$$

Here we multiply the whole number 16, by the numerator 2, divide the product by the denominator 3, and we obtain the result $10\frac{2}{3}$.

3. *Explanation.* Multiplying a whole number by the numerator of a fraction is equivalent to multiplying the numerator by a whole number, the effect of which is explained under Prop. I. Dividing the product by the denominator is merely to change it to a mixed number (Case III.).

4. *Examples.*

(1.) Multiply $\frac{2}{3}$ by 36.	Ans. 24.
(2.) Multiply 28 by $\frac{1}{3}$.	Ans. $9\frac{1}{3}$.
(3.) Multiply $\frac{3}{4}$ by 48.	Ans. 36.
(4.) Multiply 325 by $\frac{5}{8}$.	Ans. $203\frac{1}{8}$.
(5.) Multiply 259 by $\frac{7}{12}$.	Ans. $151\frac{1}{12}$.
(6.) Multiply $\frac{9}{20}$ by 20.	Ans. 9.
(7.) Multiply $\frac{18}{45}$ by 45.	Ans. 18.
(8.) Multiply 312 by $\frac{298}{312}$.	Ans. 298.

§ 77. 1. CASE XI. *To multiply a fraction by a fraction.*

RULE. Multiply the numerators together, and the denominators together, and the products will be respectively the numerator and denominator of the required product.

2. *Illustration.* Multiply $\frac{3}{4}$ by $\frac{4}{5}$.

$$\begin{array}{r|l} 1\ \cancel{4} & 3 \\ 5 & \cancel{4}\ 1 \\ \hline 5 & 3 = \frac{3}{5}. \end{array}$$

Here we write the denominators at the left and the numerators at the right of a perpendicular line; then we cancel, so far as we can, by which we dispose of 4 on each side of the line; we then have 5 and 1 on the left to be multiplied together, and on the right 3 and 1; neglecting the 1 on each side, which we may do without varying the result, because a number multiplied by unity gives the same, and we have 5 as the denominator and 3 as the numerator of the required product.

3. *Explanation.* By Proposition I. we see that multiplying the numerator alone increases the value of the fraction, by taking too many of the same parts; and by Proposition III. we see that multiplying the denominator diminishes the value of the fraction, by increasing the number of parts necessary to make unity.

It results then that the multiplication of a fraction by a fraction increases the number of its parts and diminishes its value. But, if the denominator of the multiplicand, or fraction to be multiplied, were multiplied by the denomi-

nator only of the multiplier, the parts would be increased as many times too much as the numerator of the multiplier denotes. The proper value of the fraction then must be restored by multiplying the numerator of the multiplicand by the numerator of the multiplier. This operation, it will be seen, is precisely that involved in Case V.

4. *Examples.*

(1.) Multiply $\frac{3}{4}$ by $\frac{2}{3}$. Ans. $\frac{1}{2}$.
(2.) Multiply $\frac{1}{9}$ by $\frac{12}{14}$. Ans. $\frac{2}{21}$.
(3.) Multiply $\frac{4}{17}$ by $\frac{5}{27}$. Ans. $\frac{20}{459}$.
(4.) Multiply $\frac{19}{42}$ by $\frac{7}{38}$. Ans. $\frac{1}{12}$.
(5.) Multiply $\frac{26}{57}$ by $\frac{2}{43}$. Ans. $\frac{52}{2451}$.
(6.) Multiply $\frac{1}{9}$ of 8 by $\frac{3}{5}$ of 7. Ans. $3\frac{11}{15}$.
(7.) Multiply $\frac{7}{9}$, $\frac{3}{5}$, and $4\frac{5}{14}$ together. Ans. $2\frac{1}{30}$.
(8.) Multiply $5\frac{2}{3}$, $\frac{6}{7}$ of $\frac{3}{5}$, and $4\frac{1}{6}$ together. Ans. $12\frac{1}{7}$.

(9.) Multiply $\dfrac{6}{\frac{1}{2}}$ by 7. Ans. 84.

(10.) Multiply $\frac{1}{3}$ of $\dfrac{5\frac{1}{2}}{9}$ by 3. Ans. $\frac{11}{18}$.

(11.) A man owning $\frac{1}{3}$ of a farm, sold $\frac{1}{2}$ of his share; what part of the farm did he sell? Ans. $\frac{1}{6}$.

(12.) What part of a foot is $\frac{2}{3}$ of $\frac{1}{12}$ of a foot? Ans. $\frac{1}{18}$.

(13.) What part of a mile is $\frac{3}{8}$ of $\frac{2}{3}$ of a mile? Ans. $\frac{1}{4}$.

(14.) What costs $\frac{3}{4}$ of a yard of cloth, at $\frac{5}{8}$ of a dollar per yard? Ans. $\frac{15}{32}$ of a dollar.

(15.) What will $2\frac{1}{2}$ tons of hay cost, at $16\frac{3}{4}$ dollars per ton? Ans. $41\frac{7}{8}$ dollars.

(16.) What will $2\frac{1}{3}$ barrels of sugar cost, at $18\frac{1}{4}$ dollars per barrel? Ans. $42\frac{7}{12}$ dollars.

(17.) What will $8\frac{3}{4}$ pounds of tea cost, at $1\frac{1}{4}$ dollars per pound? Ans. $10\frac{15}{16}$ dollars.

(18.) What will $4\frac{3}{4}$ cords of wood cost, at $3\frac{3}{4}$ dollars per cord? Ans. $17\frac{13}{16}$ dollars.

(19.) What will $12\frac{1}{2}$ barrels of sugar cost, at $15\frac{1}{4}$ dollars per barrel? Ans. $190\frac{5}{8}$ dollars.

(20.) What will $7\frac{3}{4}$ yards cost, at $\frac{5}{6}$ of a dollar per yard?

§ 78. To find the difference between fractions, we first change them to their least common denominators, as for Addition, and then subtract the numerators as we subtract whole numbers. Hence we have,

§ 79. 1. CASE XII. *To subtract Vulgar Fractions.*

RULE. Change the fractions to their least common denominator, subtract the numerators, and, under the remainder, write the denominator.

2. *Illustration.* Subtract $\frac{8}{12}$ from $\frac{3}{4}$.

$$\frac{3}{4} - \frac{8}{12}$$

$$\begin{array}{r|l|l} 4 & 3 & 8 \\ 3\ \cancel{12} & \cancel{12}\ \ 3 & \cancel{4} \\ \hline 12 & 9\ - & 8 = \frac{1}{12}. \end{array}$$

(1.) Here we first change the fractions to their least common denominator by canceling as far as it can be done; we thus obtain as the least common denominator, 12, and as numerators 9 and 8. We then take the numerator of the subtrahend 8, from the numerator of the minuend 9, and we have a remainder 1, under which we write the least common denominator 12, and we have the required remainder $\frac{1}{12}$.

(2.) There is no principle involved in this operation which has not already been fully explained. Explanation is therefore unnecessary.

(3.) For this operation, mixed numbers should be changed to improper fractions, and compound fractions to single ones.

3. *Examples.*

(1.) From $\frac{3}{4}$ take $\frac{5}{7}$. Ans. $\frac{1}{28}$.
(2.) From $\frac{5}{6}$ take $\frac{7}{12}$. Ans. $\frac{3}{12}$.
(3.) From $\frac{13}{30}$ take $\frac{2}{5}$ of $\frac{7}{12}$. Ans. $\frac{1}{5}$.
(4.) From $\frac{33}{35}$ take $\frac{13}{14}$. Ans. $\frac{1}{70}$.
(5.) From $\frac{4}{7}$ take $\frac{3}{7}$ of $\frac{2}{3}$. Ans. $\frac{2}{7}$.
(6.) From $\frac{3}{4}$ take $\frac{2}{7}$ of $\frac{5}{8}$. Ans. $\frac{4}{7}$.
(7.) From $\frac{49}{50}$ take $\frac{5}{9}$. Ans. $\frac{191}{450}$.
(8.) From $37\frac{1}{4}$ take $19\frac{4}{7}$. Ans. $17\frac{19}{28}$.

(9.) From $13\frac{1}{13}$ take $11\frac{1}{5}$. Ans. $1\frac{57}{65}$.

(10.) From $\frac{290}{216}$ take $\frac{7}{144}$. Ans. $\frac{559}{864}$.

(11.) From $\frac{8}{15}$ take $\frac{9}{20}$. Ans. $\frac{1}{12}$.

(12.) From $\frac{5}{6}$ take $\frac{4}{5}$. Ans. $\frac{1}{30}$.

(13.) In an orchard $\frac{1}{2}$ of the trees bear apples, $\frac{1}{4}$ peaches, $\frac{1}{6}$ plums, 30 pears, 15 cherries, and 5 quinces: how many trees are there in the orchard? Ans. 600 trees.

(14.) $\frac{1}{2}$, and $\frac{1}{3}$ of a school, and 10 scholars make up the school: how many scholars are there in it?

Ans. 60 scholars.

(15.) There is an army to which if you add $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ itself, and take away 5000, the sum total will be 100,000: what is the number of the whole army?

Ans. 50400 men.

(16.) If a pole stand $\frac{1}{5}$ in the mud, $\frac{3}{8}$ in the water, and the rest above water: what part is above water? Ans. $\frac{17}{40}$.

(17.) A person spent $\frac{1}{5}$ of his life at school, $\frac{1}{3}$ in America, and the rest in Europe: how much of his life did he spend in Europe? Ans. $\frac{7}{15}$.

§ 80. The division of a fraction by a whole number, a whole number by a fraction, and a fraction by a fraction, are simple operations consequent on Propositions II. and III.

If a mixed number occur in either the dividend or divisor, or both, change it to an improper fraction; if a compound fraction, change it to a single one. Hence we have,

§ 81. 1. CASE XIII. *To divide a fraction by a whole number.*

RULE. Multiply the denominator; or, when it can be done, divide the numerator.

2. *Illustrations.*

(1.) $\frac{3}{4}\times3=\frac{3}{12}=\frac{1}{4}$. or, (2.) $\frac{3}{4}\div3=\frac{1}{4}$.

Here, in the first illustration, we multiply the denominator 4 by 3, and we obtain the result $\frac{3}{12}$, which equals $\frac{1}{4}$.

In the second illustration, we divide the numerator 3 by 3, and we obtain the result $\frac{1}{4}$.

3. *Explanation.* The explanations of Propositions II. and III. are the explanations of this operation.

4. *Examples.*

(1.) Divide $\frac{18}{21}$ by 9. Ans. $\frac{2}{21}$.
(2.) Divide $\frac{13}{9}$ by 11. Ans. $\frac{13}{99}$.
(3.) Divide $\frac{21}{12}$ by 2. Ans. $\frac{7}{8}$.
(4.) Divide $\frac{81}{97}$ by 9. Ans. $\frac{9}{97}$.
(5.) Divide $\frac{1}{53}$ by 8. Ans. $\frac{1}{424}$.
(6.) Divide $\frac{30}{12}$ by 6. Ans. $\frac{5}{12}$.
(7.) Divide $\frac{19}{8}$ by 40. Ans. $\frac{19}{320}$.
(8.) Divide $\frac{1}{6}$ by 72. Ans. $\frac{1}{432}$.
(9.) Divide $\frac{47}{90}$ by 856. Ans. $\frac{47}{77040}$.

§ 82. 1. CASE XIV. *To divide a whole number by a fraction.*

RULE. Multiply the whole number by the denominator of the fraction, and divide the product by the numerator.

2. *Illustration.* Divide 9 by $\frac{3}{5}$.

$$9 \div \tfrac{3}{5} = \begin{array}{r|l} & \cancel{9}\ 3 \\ 1\ \cancel{3} & 5 \\ \hline 1 & 15 = \tfrac{15}{1} = 15. \end{array}$$

Here we write the numbers which the rule directs to be multiplied together under each other, with a perpendicular line between them; the number to form the denominator, as usual, to the left. Then, canceling as far as we are able, we obtain the result 15; that is, $\frac{3}{5}$ is contained in 9, 15 times.

3. *Explanation.* The whole number is first multiplied by the denominator of the divisor to change it to a fraction with that denominator (§ 55. 1). The division then, is that of finding how many times one fraction is contained in another of the same name, or denoting the same parts of unity. The dividend and divisor, therefore, being fractions, the expression is in effect a complex fraction to be changed to a single one (§ 71. 1).

4. The division of 9 by $\frac{3}{5}$ fully carried out would be,

$$\frac{9}{\frac{3}{5}}=\frac{\frac{9\times 5}{5}}{\frac{3}{5}}=\frac{\frac{45}{\cancel{5}}}{\frac{3}{\cancel{5}}}=\frac{45}{3}=15, \text{ the same as above.}$$

5. *Examples.*

(1.) Divide 18 by $\frac{3}{7}$. Ans. 42.
(2.) Divide 63 by $\frac{9}{13}$. Ans. 91.
(3.) Divide 63 by $\frac{9}{11}$. Ans. 77.
(4.) Divide 66 by $\frac{2}{3}$. Ans. 99.
(5.) Divide 32 by $\frac{4}{5}$. Ans. 40.
(6.) Divide 472 by $\frac{8}{9}$. Ans. 531.
(7.) Divide 563 by $\frac{4}{6}$. Ans. $844\frac{1}{2}$.
(8.) Divide 47 by $\frac{12}{14}$. Ans. $54\frac{5}{6}$.
(9.) Divide 94 by $\frac{5}{6}$. Ans. $112\frac{4}{5}$.

§ 83. 1. CASE XV. *To divide a fraction by a fraction.*

RULE. Write the divisor under the dividend as a complex fraction; change that fraction to a single one, and the result will be the quotient required.

2. *Illustration.* Divide $\frac{5}{6}$ by $\frac{3}{4}$.

$$\tfrac{5}{6}\div\tfrac{3}{4}=\frac{\frac{5}{6}}{\frac{3}{4}}=\frac{\frac{5}{6}\times 4}{\frac{3}{4}\times 6}=\frac{\frac{20}{24}}{\frac{18}{24}}=\tfrac{20}{18}=\tfrac{10}{9}.$$

Or, shorter, $\frac{5}{6}\div\frac{3}{4}=\dfrac{\frac{5}{6}\times 4}{\frac{3}{4}\times 6}=\dfrac{20}{18}$ or,

$$\begin{array}{r|l|l} \multicolumn{3}{c}{\tfrac{5}{6}\quad\div\quad\tfrac{3}{4}} \\ 6 & 5 & 3 \\ 2\cancel{4} & \cancel{4}2 & \cancel{6}3 \\ \hline 12 & 10\ \div & 9=\tfrac{10}{9}. \end{array}$$

3. *Explanation.* This is, in effect, multiplying the numerator of the dividend into the denominator of the divisor for the required numerator, and the denominator of the dividend into the numerator of the divisor for the required denominator, the mode prescribed in most treatises on the subject. It is a mode, however, which requires a labored

explanation to make it intelligible to the pupil. We, therefore, prefer that here presented. The principle of this is clear and simple, and with it the pupil is already familiar (§ 71. 1, 2, 3, 4).

4. *Examples.*

(1.) Divide $\frac{10}{24}$ by $\frac{5}{8}$. Ans. $\frac{2}{3}$.
(2.) Divide $\frac{16}{90}$ by $\frac{8}{5}$. Ans. $\frac{1}{9}$.
(3.) Divide $\frac{64}{111}$ by $\frac{523}{1213}$. Ans. $1\frac{19579}{58053}$.
(4.) Divide $\frac{2}{6}$ by $\frac{6}{12}$. Ans. $\frac{2}{3}$.
(5.) Divide $\frac{21}{32}$ by $\frac{2}{3}$. Ans. $\frac{63}{64}$.
(6.) Divide $\frac{7}{8}$ by $\frac{2}{3}$. Ans. $1\frac{5}{16}$.
(7.) Divide $\frac{8}{9}$ by $\frac{1}{2}$. Ans. $1\frac{7}{9}$.
(8.) Divide $\frac{1}{3}$ of $\frac{5}{6}$ by $\frac{3}{5}$ of $\frac{1}{4}$. Ans. $1\frac{23}{27}$.
(9.) Divide $\frac{4}{5}$ of $\frac{6}{14}$ of $\frac{3}{4}$ by $\frac{1}{4}$ of $\frac{6}{7}$ of $\frac{9}{10}$. Ans. $1\frac{1}{3}$.
(10.) Divide $\frac{15}{17}$ of $\frac{11}{12}$ by $\frac{3}{5}$ of $\frac{6}{7}$. Ans. $1\frac{701}{1224}$.
(11.) $\frac{3}{5}$ of 23 less $\frac{7}{10}$ of 4 are how many times $3\frac{1}{7}$?
Ans. $3\frac{1}{2}$.

(12.) $\frac{4}{9}$ multiplied by $7\frac{1}{3}$ and the product divided by $\frac{2}{5}$ gives what result? Ans. $8\frac{4}{27}$.

(13.) How much cloth, at $1\frac{1}{2}$ dollar (that is, $\frac{3}{2}$ dollars) a yard, can be bought for $4\frac{1}{5}$ dollars? Ans. $2\frac{4}{5}$ yards.

(14.) A man distributed $8\frac{1}{2}$ bushels of wheat among some poor persons, giving $1\frac{1}{2}$ bushel to each; how many did he give it to? $5\frac{2}{3}$ persons.

(15.) If a soldier is allowed $1\frac{1}{3}$ pound (that is, $\frac{4}{3}$ of a pound) of meat in a day, to how many soldiers would $6\frac{5}{8}$ pounds be allowed? Ans. $4\frac{31}{32}$ soldiers.

(16.) If $1\frac{2}{3}$ ton of hay will keep a horse through the winter, how many horses will $10\frac{1}{2}$ tons keep?
Ans. $6\frac{3}{10}$ horses.

(17.) At $2\frac{1}{3}$ dollars a box, how many boxes of raisins can be bought for $10\frac{4}{7}$ dollars? Ans. $4\frac{26}{49}$ boxes.

(18.) At $1\frac{3}{5}$ dollar a pound, how many pounds of indigo can be bought for $9\frac{3}{9}$ dollars? Ans. $5\frac{5}{6}$ pounds.

(19.) At $1\frac{4}{7}$ dollar a barrel, how many barrels of raisins can be bought for $9\frac{3}{5}$ dollars? Ans. $6\frac{6}{55}$ barrels.

(20.) At $\frac{7}{8}$ of a dollar a piece, how many pieces of nankin can be bought for $8\frac{3}{4}$ dollars? Ans. 10 pieces.

(21.) At $\frac{5}{7}$ of a dollar a pound, how many pounds of tea can be bought for $7\frac{2}{9}$ dollars? Ans. $10\frac{1}{9}$ pounds.

DECIMAL FRACTIONS.

§ 84. 1. In notation of whole numbers, it has been seen that the local value of figures varies according to the place in which they stand (§ 5, 3), that every figure designates a quantity ten times greater for every place it is removed to the left from that of simple units (7, 5). The increase from right to left was, therefore, *tenfold* (§7. 5).

2. Now, it is obvious, that if we wish to denote quantities less than unity, otherwise than by Vulgar Fractions, so that the notation of the numbers expresing them may correspond with that of whole numbers, we should write them in places below unity, that is to the right of unity, that being the lowest place in whole numbers, and make their decrease from unity to the right *tenfold.*

3. On such a system the first place to the right of unity would be *tenths* of units, the second place *hundredths*, the third *thousandths*, and so on.

4. Thus, there would be implied below each figure a denominator of 10, 100, 1000, or 1 with such a number of ciphers annexed as would denote the place occupied by the figure acording to the notation.

5. It would then be desired to separate by some mark or sign between the parts thus written and the whole numbers.

This might be done by a simple point, or comma (,), which, from the end it would answer, might be called a decimal point, or separatrix.

6. Such a method of denoting parts we have ; and differing from vulger fractions in having the denominator implied, not expressed, and this determined by place in notation ; the notation being from left to right, by a fixed decrease, in tenths, or tenfold, it is called *decimal fractions.*

7. Therefore, a *decimal fraction* is a fraction which is expressed by writing the numbers denoting the parts at the right of a point, or comma, called a decimal point, or separating.

8. *Illustration.*

	Tenths.	Hundredths.	Thousandths.	Tens of Thousandths.	Hundreds of Thousandths.	Millionths.	Tens of Millionths.	Hundreds of Millionths.	Billionths.	Tens of Billionths.	Hundreds of Billionths.	Trillionths.
,	4	6	1	5	8	9	7	5	1	0	3	1

The next place to the right would be Tens of Trillionths; the next, Hundreds of Trillionths, and so on.

9. The names used in Notation of whole numbers are used in succession from left to right, with only the termination changed by the addition of *th*, or *ths*, at the end.

10. To each of these figures there is implied a denominator, corresponding to the name of its place.

11. This denominator is always 1, with as many ciphers annexed as there are figures in the numerator. To the 4, at the left in the above, the denominator implied is 10; to the 6, 100; to the 4, 1000; and so on.

12. Decimals are read by calling each figure from left to right separately, naming the part which it denotes of its implied denominator; as, in the above, four tenths, six hundredths, one thousandth, and so on; or, by calling them off together as whole numbers, naming the implied denominator to the last figure, as the denominator to the whole; as four hundred and sixty-one Billions, five hundred and eighty-nine Millions, seven hundred and fifty-one Thousand, and thirty-one Trillionths; or, shorter, mentioning the (,) point, and then reading the subsequent numbers simply as they occur, leaving the denominators out, as understood from the principles of the system.

13. As in Vulgar Fractions we write a fraction to a whole number and form a mixed number (§ 56.), so in Decimals. Thus, 4,5; 89,785.

14. *Examples.*

(1.) Seventy-six hundredths.
(2.) Forty-nine thousand, four hundred, and nine tenths.
(3.) Sixty-four trillions, and thirty-one millionths.
(4.) Five hundred and seventy, and six tenths.
(5.) Four hundred, and nine ten thousandths.
(6.) Twenty, and two hundredths.
(7.) Eighty-one hundred, and four ten thousandths.
(8.) One million, one thousand, and one billionth.
(9.) Seventy-three billions, and two tenths.
(10.) Eight hundred thousand, and twenty-five thousandths.
(11.) Eleven, and seven billionths.
(12.) Four hundred and twenty-one, and nineteen thousandths.

15. From the nature of decimals, we deduce four fundamental propositions.

§ 85. 1. Proposition I. A cipher prefixed to a decimal decreases its value tenfold.

2. *Illustration.* To ,5 prefix 0. ,05.

Here the expression $,5=\frac{5}{10}=\frac{1}{2}$; but $,05=\frac{5}{100}=\frac{1}{20}$.

3. *Explanation.* By prefixing ciphers to a decimal, the figure or figures previously composing it are made to stand as many places farther to the right of the separatrix as there are ciphers prefixed; and, consequently, their value is decreased so many times tenfold (§ 84. 6).

4. *Examples.*

(1.) Change ,5 to five thousandths.
(2.) Change ,003 to three millionths.
(3.) Change ,017 to seventeen billionths.
(4.) Change ,84 to eighty-four trillionths.

§ 86. 1. Proposition II. A cipher annexed to a decimal does not alter its value.

2. *Illustration.* To ,5 annex 0. ,50.

Here the expression $,5=\frac{5}{10}=\frac{1}{2}$; and $,50=\frac{50}{100}=\frac{1}{2}$.

3. *Explanation.* This operation corresponds with that of prefixing ciphers to a whole number, which does not vary the value of that whole number, because the increase being from right to left, and the places being so named, a place occupied by a cipher, which has no value (§ 4. 3), would be read as nothing of that place, or disregarded.

So, here, the decrease being from left to right, the succeeding places can have significancy only as they are occupied by significant figures; for if, at the end, at the right, there stand a cipher, and it were named in connection with its place, it would be read as nothing of that place, or disregarded.

4. *Examples.*

(1.) Change ,8 to eighty hundredths.
(2.) Change ,046 to four hundred and sixty thousandths.
(3.) Change ,74 to seventy-four thousand ten hundred thousandths.
(4.) Change ,9 to nine hundred thousandths.

§ 87. 1. Proposition III. Removing the decimal point one place to the left, decreases the fraction tenfold; or divides it by 10.

2. *Illustration.* 43,752. The decimal point removed one place to the left, gives 4,3752.

3. *Explanation.* The explanation to Prop. I. suffices for this; for the principle, it is obvious, is the same in each; or, we may add, that the removing of the point to the left puts at its right in the place of decimals, and so makes decimals, so many of the whole numbers as by the removal of the point are brought to its right.

4. *Examples.*

(1.) Diminish ,15 a thousandfold.
(2.) Diminish 413,568 a millionfold.
(3.) Diminish 67 a hundredfold.

(4.) Diminish 8 tenfold.
(5.) Diminish 6789 tenfold.
(6.) Diminish 3,934 a hundredfold.

§ 88. 1. PROPOSITION IV. Removing the decimal point one place to the right increases the fraction tenfold, or multiplies it by 10.

2. *Illustration.* ,4673. The decimal point removed one place to the right gives 4,673.

3. *Explanation.* This Proposition is the converse of the last; as the removal of the point to the left decreased the fraction tenfold, bringing it back again or removing it to the right, it is evident, must increase it, or multiply it so many fold; or, we may add, that the removing of the point to the right puts at its left, in the place of whole numbers, and so makes whole numbers, so many of the decimals as by the removal of the point are brought to its left.

4. *Examples.*

(1.) Increase ,63 tenfold.
(2.) Increase ,9 a hundredfold.
(3.) Increase 7,541 a thousandfold.
(4.) Increase ,81643 ten thousandfold.
(5.) Increase 314,8765 a millionfold.
(6.) Increase 12,4 tenfold.

§ 89. 1. From the above, it will be easy to perceive the advantage, which decimal fractions have over vulgar fractions; all the multiplications and divisions, which are performed by the denominator of the latter, are performed with respect to the former, by the addition or suppression of a number of ciphers, or by simply changing the place of the comma.

2. It is evident, too, by decimals thus developed, that the remainder which at the close of an operation in Division was too small to be divided by the divisor, and the division of which we were obliged merely to indicate, and which we treated (§ 45. 1, 2) as the origin of fractions, may be divided and a more exact result be obtained.

3. To do this, we have only to annex ciphers for deci-

mals, and divide according to the law for the division of whole numbers, observing to distinguish as decimals the quotient thus obtained.

§ 90. 1. CASE I. *To change a vulgar fraction to a decimal.*

RULE. Write a decimal point at the right of the numerator, suppose a cipher or ciphers annexed, divide by the denominator, and the quotient will be the decimal required.

2. *Illustration.* Change $\frac{1}{4}$, $\frac{3}{8}$ $\frac{3}{7}$ and $\frac{1}{3}$ to decimal fractions.

4)1,0(,25		$\frac{3}{8}$=,375.
8		
—	or $\frac{1}{4}$=,25.	$\frac{3}{7}$=,42857142+.
20		$\frac{1}{3}$=,33333+.
20		

(1.) In changing $\frac{1}{4}$ we suppose a cipher annexed to the numerator 1, after the point, and then divide as in common division. The quotient we obtain is 25, which, being a decimal, we preceded by the decimal point. The quotient is then 25 hundredths.

(2.) In changing $\frac{3}{8}$, we proceed precisely as in changing $\frac{1}{4}$.

(3.) In changing $\frac{3}{7}$ we proceed, as far as is done in the illustration, as with $\frac{1}{4}$, and $\frac{3}{8}$; but, when the 1 in the quotient is obtained, we find our remainder to be 3, the same number for a dividend with which we started; and we have then a repetition of the quotient figures previously obtained—42—and should have the others did we continue the division. Knowing thus what would be the figures for the quotient, we may write them as far as is neecessary without dividing, and when we desire to carry them no further, we annex +, plus, to show that the result is not exact, that more might yet be obtained.

Such series of recurring numbers are called repeating or circulating decimals.

(4.) In changing $\frac{1}{3}$, we constantly obtain from the first the same quotients, and the same remainders; the operation need not therefore be continued.

(5.) The further such a series is carried, the nearer is the approach to an exact result; though such result could never be obtained. ,33 is nearer to unity than ,3; ,333 than ,33, and ,3333 than ,333, and so on; but by no approaches of this kind could we arrrive at unity.

(6.) This approaching to a true result by decimals, is called *approximating*. It is sufficient usually that it be carried to four or five places.

3. *Explanation*. Changing a Vulgar Fraction to a decimal is in effect changing a fraction of one denomination to another (52. 1). This is done by multiplying the numerator of the given fraction by the denominator of the required one, and dividing the product by the denominator of the given fraction, the quotient being the numerator of the fraction required.

4. *Examples.*

(1.) Change $\frac{1}{2}$ to a decimal.	Ans. ,5.
(2.) Change $\frac{3}{4}$ to a decimal.	Ans. ,75.
(3.) Change $\frac{4}{5}$ to a decimal.	Ans. ,8.
(4.) Change $\frac{7}{8}$ to a decimal.	Ans. ,875.
(5.) Change $\frac{2}{9}$ to a decimal.	Ans. ,222+.
(6.) Change $\frac{5}{6}$ to a decimal.	Ans. ,8333+.
(7.) Change $\frac{15}{16}$ to a decimal.	Ans. ,9375.
(8.) Change $\frac{12}{13}$ to a decimal.	Ans. ,923+.
(9.) Change $\frac{4}{25}$ to a decimal.	Ans. ,16.
(10.) Change $\frac{211}{250}$ to a decimal.	Ans. ,844.
(11.) Change $\frac{3}{5}$ of $\frac{2}{3}$ to a decimal.	Ans. ,4.
(12.) Change $\frac{2}{11}$ to a decimal.	Ans. ,181818+.
(13.) Change $\frac{7}{16}$ to a decimal.	Ans. ,4375.
(14.) Change $\frac{17}{25}$ to a decimal.	Ans. ,68.
(15.) Change $\frac{437}{821}$ to a decimal.	Ans. ,53227+.
(16.) Change $\frac{18}{19}$ to a decimal.	Ans. ,94734+.
(17.) Change $\frac{5}{11}$ to a decimal.	Ans. ,4545+.
(18.) Change $\frac{2}{12}$ to a decimal.	Ans. ,1666+.

§ 91. 1. CASE II. *To change a decimal to a vulgar fraction.*

RULE. Write under the decimal its implied denominator, and omit the point.

2. *Illustration.* The vulgar fraction for ,25 is $\frac{25}{100}=\frac{1}{4}$; for ,75 is $\frac{75}{100}=\frac{3}{4}$.

3. *Explanation.* This operation is merely supplying the implied denominator, and then bringing the fraction to its lowest terms.

4. *Examples.*

(1.) Change ,75 to a vulgar fraction. Ans. $\frac{3}{4}$.
(2.) Change ,875 to a vulgar fraction. Ans. $\frac{7}{8}$.
(3.) Change ,1875 to a vulgar fraction. Ans. $\frac{3}{16}$.
(4.) Change ,0005 to a vulgar fraction. Ans. $\frac{1}{2000}$.
(5.) Change ,00125 to a vulgar fraction. Ans. $\frac{1}{800}$.
(6.) Change 6,015 to a vulgar fraction. Ans. $6\frac{3}{200}$.
(7.) Change ,9375 to a vulgar fraction. Ans. $\frac{15}{16}$.
(8.) Change ,16 to a vulgar fraction. Ans. $\frac{4}{25}$.
(9.) Change ,844 to a vulgar fraction. Ans. $\frac{211}{250}$.
(10.) Change ,4375 to a vulgar fraction. Ans. $\frac{7}{16}$.
(11.) Change ,68 to a vulgar fraction. Ans. $\frac{17}{25}$.

5. If the decimal to be changed be a repeating decimal, *make the figures repeated a numerator, and write the same number of 9s under it for a denominator.*

6. If to common decimals, repeating decimals are annexed, and they are required to be changed, first, find the value of the repeating decimals (§ 91. 5), then of the other decimals, and add together the results.

7. It is a common practice, instead of writing the repeating figures several times, to write a dot over the repeating figure, when but one is repeated, and, over the first and last, when more than one is repeated. Thus, $\dot{3}$ denotes that the 3 is to be continually repeated; and $\dot{2}3\dot{4}$ denotes that 234 is to be continually repeated.

8. *Examples.*

(1.) Reduce $0,\dot{1}\dot{8}$ to the form of a vulgar fraction.
Ans. $\frac{2}{11}$.

(2.) Reduce $0,\dot{7}\dot{2}$ to the form of a vulgar fraction.
Ans. $\frac{8}{11}$.

(3.) Change $0,8\dot{3}$ to a vulgar fraction.

Ans. $\frac{5}{6}$.

(4.) Change $0,2\dot{4}1\dot{8}$ to a vulgar fraction.

Ans. $\frac{1208}{4995}$.

(5.) Change $0,27\dot{5}46\dot{3}$ to a vulgar fraction.

Ans. $\frac{22953}{83325}$.

(6.) Change $0,91\dot{6}$ to a vulgar fraction.

Ans. $\frac{11}{12}$.

§ 92. The operation of addition differs from that in whole numbers only in writing the numbers and the point in the sum.

§ 93. 1. CASE III. *To add decimal fractions.*

RULE. I. Write the fractions under each other, in the order of tenths under tenths, hundredths under hundredths, and so on.

II. If there are whole numbers prefixed, write them as in Addition of whole numbers (§ 16. 1).

III. Begin at the right to add, and proceed in all respects as in Addition of whole numbers.

IV. Write the point in the sum between the decimals and whole numbers, directly under the points above.

2. *Illustration.*

$$\begin{array}{r@{,}l} 807 & 2659 \\ 70 & 602 \\ 4 & 06 \\ 151 & 7 \\ \hline 1033 & 6279 \end{array}$$

The steps taken here are apparent and need not be detailed.

3. *Explanation.* Decimals, like whole numbers, increasing from right to left tenfold, it is evident that their addition must be on the same principle as the addition of whole numbers (§ 16. 3), that the carrying will be constantly by 10, and that when we add the last or left hand

column of decimals, if there be any number to carry, it must be taken to the left of the place for the point, that is to that for whole numbers; and, therefore, that the point in the sum must have the same relative place which it occupied in the numbers added.

4. *Examples.*

(1.) Add 6,07+48,903+563,1+1000,241.
Ans. 1618,314.

(2.) Add 409,903+107,7842+6,1043+10,2074.
Ans. 533,9989.

(3.) Add 4,003+54,9+3,21+6,7202.
Ans. 68,8332.

(4.) Add one hundred, and one thousandth+thirty-four, and six ten thousandths+and one thousand and three and eight hundred and seventy-nine thousand and twenty-five millionths.
Ans. 1137,880625.

§ 94. The operation of multiplication corresponds in all respects with the multiplication of whole numbers, except in writing the point in the product.

§ 95. 1. CASE IV. *To multiply decimal fractions.*

RULE. I. Write the multiplier under the multiplicand, as for the multiplication of whole numbers.

II. Multiply as in multiplication of whole numbers.

III. Write the point in the product to the left of as many figures as there are decimals in both factors.

IV. If there are not so many figures in the product as there are decimals in both factors, prefix ciphers to make up the deficiency.

2. *Illustration.*

```
    172,84
    36,003
   -------
     51852
  103704
  51852
 ---------
6222,75852
```

(1.) Here we write the numbers and multiply precisely as in whole numbers.

(2.) To write the point in the product we first notice the decimal places in the multiplicand, which we see to be two, and as we disregarded the point at their left in the multiplication, and treated the multiplicand as a whole number, it is evident, in multiplying it, we multiplied a number one hundred times too great. The product, therefore, being on that account one hundred times too great, we suppose it decreased by dividing it by 100, or the separation by a point of two figures at its right.

(3.) We then notice the decimal places in the multiplier, which we see to be three, and as we disregarded the point at their left in the multiplication, and treated the multiplier as a whole number, it is evident, in multiplying by it, we multiplied by a number one thousand times too great. The product on that account being still one thousand times too great, we suppose it decreased by dividing it by 1000 more, or the separation at the right of three figures more by a point three places further to the left.

(4.) The figures, then, at the left of which the point is to be written, we find to be five; corresponding to the number of decimals in both the factors. Therefore, at the left of the 7, we write the point.

3. *Explanation.* The multiplication of decimals is like that of whole numbers, for the same reason that their addition and subtraction are the same; because of the correspondence of their systems of notation (§ 84. 2). The only matter in this operation needing explanation relates to the writing of the point in the product. We direct in the rule that it be written at the left of as many figures as there are decimals in both factors.

If there were decimals in but one factor, in the multiplicand for instance, and the point were disregarded, that factor would be multiplied by so many times tenfold as there were decimals thus made whole numbers (§ 88. 3). The product therefore would be so many times too great. It would then be necessary to decrease it so many times, by writing the point at the left of so many figures from its right (§ 87. 3).

If there were decimals in the other factor, the multiplier

also, and the point were disregarded, that factor would be increased so many times tenfold as there were decimals thus made whole numbers (§ 88. 3), and it would consequently, in the multiplication, increase the multiplicand so many times more than was required. The product therefore would be so many times too great. It would then be necessary to decrease it so many times, by making decimals of so many places from its right more than was done for the decimals in the multiplicand, by writing the point so many places further to the left.

If so many places are not contained in the product, it is evident that it would express too great a result if the point were written directly at its left. Therefore, ciphers are prefixed to throw the point to the left to decrease the product sufficiently to make it a true result.

4. *Examples.*

(1.) Multiply six, and seven hundredths, by forty-eight and nine hundred and three thousandths.

Ans. 296,84121.

(2.) Multiply 231,415 by 8.	Ans. 1851,320.
(3.) Multiply 32,1509 by 15.	Ans. 482,2635.
(4.) Multiply ,840 by 840.	Ans. 705,600.
(5.) Multiply 1,236 by 13.	Ans. 16,068.
(6.) Multiply 223,86 by 2,500.	Ans. 559,65000.
(7.) Multiply 35,640 by 26,18.	Ans. 933,05520.
(8.) Multiply 8,4960 by 1,618.	Ans. 13,7465280.
(9.) Multiply ,5236 by ,2808.	Ans. 0,14702688.
(10.) Multiply ,11785 by ,27.	Ans. 0,0318195.

§ 96. The subtraction of decimals corresponds with the subtraction of whole numbers, except when the decimals in the subtrahend are lower than those in the minuend, and in writing the numbers and the point in the remainder.

§ 97. 1. Case V. *To subtract decimal fractions.*

Rule. I. Write the subtrahend under the minuend, in the order of tenths under tenths, hundredths under hundredths, and so on.

II. If there are whole numbers prefixed, write them as in subtraction of whole numbers.

III. Begin at the right, and subtract, when there are

figures over those of the subtrahend, as in subtraction of whole numbers.

IV. When there are lower decimals in the subtrahend than in the minuend, so that some figures in the subtrahend have none over them in the minuend, suppose ciphers to be written, as decimals, to fill out the places in the minuend, and subtract from them as in subtraction of whole numbers (§ 33. Rule).

V. Write the point in the remainder between the decimals and whole numbers, directly under the points above.

2. *Illustration.*

(1.)		(2.)	
	0,9832		3,490864
	0,4986835		2,74962
	0,4845165		0,741244

(1.) In the first illustration, for the subtraction of the first three figures 835, we suppose ciphers to stand over them, and take one from the figure next above them, suppose it added to them and proceed as in subtraction of whole numbers.

(2.) In the second illustration, there being no number to subtract from the first figure 4, we write it unchanged in the remainder, and proceed with the other figures as in subtraction of whole numbers.

3. *Explanation.* It is evident from the notation of each that the principles of subtraction of whole numbers and decimals precisely correspond; and therefore, that the operation must be the same, and the result express the difference between the parts of unity exactly similar; so that the point must have the same relative place in the remainder which it occupies in the given numbers.

4. *Examples.*

(1.) From 54,09 take 36,75. Ans. 17,34.

(2.) From 763,278 take 85,39. Ans. 677,888.

(3.) From one thousand six hundred and eighteen and three hundred and fourteen thousanths, take five hundred and sixty-three and two hundred and forty-one thousandths. Ans. 1055,073.

(4.) From 304,567 take 158,632. Ans. 145,935.
(5.) From 215,003 take 1,1034. Ans. 213,8996.
(6.) From 1 take ,9993. Ans. 0,0007.
(7.) From 68,8333 take ,00042. Ans. 68,83288.
(8.) From 462,3 take 218,15. Ans. 244,15.
(9.) From 16,705 take 7,6845. Ans. 9,0205.
(10.) From 132,4 take 36,36. Ans. 96,04.
(11.) From 127,05 take 66,006. Ans. 61,044.
(12.) From 100,001 take 77,77. Ans. 22,231.
(13.) From 846,358 take 725,642. Ans. 120,716.
(14.) From five hundred and thirty-six and fifteen hundredths, take two hundred and thirty-six and eighteen hundredths. Ans. 300,132.

§ 98. The division of decimals differs from the division of whole numbers only in certain preparations of the dividend, and in writing the point in the quotient.

§ 99. 1. CASE. VI. *To divide decimal fractions.*

RULE. I. Write the divisor and dividend, and, if the dividend will contain the divisor, divide as in whole numbers.

II. If the dividend will not contain the divisor, annex ciphers, and write a cipher for every attempted division in the quotient.

III. If the divisor has in it decimals, and the dividend has not, annex as many ciphers to the dividend as there are decimals in the divisor.

IV. Write the point at the left of as many figures in the quotient as the decimals in the dividend exceed those in the divisor.

V. If there are not so many figures in the quotient as the decimal places in the dividend exceed those in the devisor, supply the deficiency by prefixing ciphers.

VI. Annex a cipher or ciphers to the remainder after the proper dividend is exhausted, if the result be not exact, and so continue, till an exact result be obtained, or one that is sufficiently exact.

VII. All the quotient figures thus obtained, distinguish as decimals.

2. *Illustration.*

4,2769÷567,432

567,432)4,276900(0,00753+
3 972024
―――――
3048760
2837160
―――――
2116000
1702296
―――――
413704

3. *Explanation.* The object of division (§ 36. 1), its principle (§ 36. 4), and the character of decimals (§ 84. 6), need only to be called to mind to render clear all the principles involved in their division.

4. When the dividend will not contain the divisor, the annexing ciphers as decimals, merely gives us a product to which the dividend is equivalent, containing so many more decimal places, and does not alter its value (§ 86. 1).

5. When the divisor has in it decimals, and the dividend has not, the division, it is evident, must be continued at least so far below unity or whole numbers, as the lowest denomination of the divisor; for the object of the division is to know into how many such parts it may be divided (§ 36. 1).

6. The point is written at the left of so many figures in the quotient as the decimal places in the dividend exceed those in the divisor, for the reason that the dividend being in effect the product of the divisor and quotient (§ 36. 3), the quotient must contain all that is wanting in the divisor to make their product equal the dividend. Now, the decimal places in the product equalling those in the two factors (§ 36. 4), it is evident that the quotient must contain as many decimals as the divisor falls short of those in the dividend; in other words, that the excess of decimals in the dividend over those in the divisor must be separated by the point from the right of the quotient.

7. The same reason, it is obvious, applies, when the decimals in the quotient do not equal the excess of those in

the dividend over those in the divisor, for prefixing ciphers to supply the deficiency. The ciphers annexed after the proper dividend has been exhausted become part of that dividend (§ 99. 4) as in the first addition of them; and for the reason above (§ 99. 6) the quotient figures thus obtained will be decimals.

4. *Examples.*

(1.) Divide 6345,925 by 54,23. Ans. 117,018+.
(2.) Divide 5673,21 by 23,0. Ans. 246,635+.
(3.) Divide 84329907 by 627,1. Ans. 134476,750+.
(4.) Divide 27845,96 by 9,8732. Ans. 2820,3581+.
(5.) Divide 200,5 by 331. Ans. 0,605742+.
(6.) Divide 563,2 by 0,057. Ans. 9880,70+.
(7.) Divide 7,25406 by 957, Ans. 0,00758.
(8.) Divide 0,00078759 by 0,525. Ans. 0,000150+.
(9.) Divide 14 by 365. Ans. 0,038356+.
(10.) Divide 14325,16 by 1,33. Ans. 10770,796+.
(11.) Divide 543,67 by 3,46. Ans. 157,13+.
(12.) If 36,34 bushels of corn grow on an acre, how many acres will produce 674 bushels? Ans. 18,5470 acres.

§ 100. 1. The divisions of the currency of the United States, correspond precisely with the divisions of decimals; therefore, the operations in it and decimals are the same.

2. The sign indicating federal money is $, and is written before the number.

3. Its denominations are the eagle, the dollar, the dime, the cent, and the mill.

Mills		marked	m.
10 Mills make one Cent,		"	¢. or ct.
10 Cents " " Dime,		"	d.
10 Dimes " " Dollar,		"	$, or doll.
10 Dollars " " Eagle,		"	E.

			¢.	m.
		d.	1 =	10.
	$ =	1 =	10 =	100.
E.	1 =	10 =	100 =	1000.
1 =	10 =	100 =	1000 =	10000.

4. Thus *one dollar* being taken as the unit, (the eagle being usually reckoned in dollars), the dime is the tenth part of a dollar; the cent is the tenth part of a dime, or the hundredth part of a dollar; and the mill is the tenth part of a cent, the hundredth part of a dime, and the thousandth part of a dollar.

5. Therefore mills are changed to dollars by writing the decimal point three places to the left, or by dividing by 1000; cents are changed to dollars by writing the decimal point two places to the left, or dividing by 100; dimes are changed to dollars by writing the decimal point one place to the left, or dividing by 10 (§ 87. 1, 3, and § 38. 3).

In all these cases, the numbers at the left of the point are the higher denominations; those at the right are in the denominations in which they were before writing the point.

6. The reverse of the above changes higher denominations to lower.

Dollars are changed to dimes by annexing one cipher; to cents, by annexing two ciphers; to mills, by annexing three ciphers; or by removing the decimal point one, two, or three places to the right (§ 86. 1, 3, and § 19. 1).

7. The coins of the United States are *three* of gold; the eagle, half-eagle, and quarter-eagle: *five* of silver; the dollar, half-dollar, quarter-dollar, dime, and half-dime; and *two* of copper; the cent and half-cent.

8. *Examples.*

(1.)	Change 1000 mills to dollars.	Ans. 1 dollar.
(2.)	Change 2500 dimes to dollars.	Ans. 250 dollars.
(3.)	Change 15 dollars to mills.	Ans. 15000 mills.
(4.)	Change 1 eagle to cents.	Ans. 1000 cents.
(5.)	Change 250 dollars to dimes.	Ans. 2500 dimes.
(6.)	Change 550 mills to dimes.	Ans. 5,50 dimes.
(7.)	Change 75 cents to mills.	Ans. 750 mills.
(8.)	Change 896 dimes to mills.	Ans. 89600 mills.
(9.)	Change 579 cents to dimes.	Ans. 57,9 dimes.
(10.)	Change 876 eagles to dollars.	Ans. 8760 dollars.
(11.)	Change 756 dimes to eagles.	Ans. 7,56 eagles.
(12.)	Change 5342 cents to eagles.	Ans. 5,342 eagles.
(13.)	Change 52 eagles to dimes.	Ans. 5200 dimes.
(14.)	Change 7982 dimes to mills.	Ans. 798200 mills.

(15.) Add together \$432,73; \$297,38; and \$333,62.
Ans. \$1063,73.

(16.) Add together \$1,55; \$0,72; \$340,89; \$0,01; \$1460,99. Ans. \$1804,16.

(17.) Add together \$72,01; \$1; \$0,01; \$0,10; \$40,70; \$560,88. Ans. \$674,70.

(18.) Add together \$101,01; \$20,15; \$42,89; \$79,81; \$41,41; \$51,51; \$38,41. Ans. \$375,19.

(19.) A grocer paid for a box of cheese \$37,21; for candles \$8,32; for a cask of wine \$7,38; for a box of raisins \$3,625; what was the whole cost?
Ans. \$56,535.

(20.) From \$1,25, take \$9,09. Ans. \$115,91.

(21.) From \$2, take \$0,05. Ans. \$1,95.

(22.) What cost 600 pounds of lard, at 15 cents per pound? Ans. \$90,00.

(23.) Bought 15 tons of hay at \$16,42 per ton; what was the whole cost? Ans. \$246,30.

(24.) What cost 349 acres of land, at \$15,49 per acre?
Ans. \$5406,01.

(25.) Bought 18 yoke of oxen, for \$72,50 per yoke; what was the whole cost? Ans. \$1305,00.

(26.) Paid \$311,70 for 15 tons of hay; what was the price per ton? Ans. \$20,78.

(27.) Paid \$658 for 280 reams of paper; what did I pay per ream? Ans. \$2,35.

(28.) Paid \$505,44 for 144 lbs. of tea; what was the price of one pound? Ans. \$3,51.

(29.) Paid \$375 for 50 firkins of butter; what was the price per firkin? Ans. \$7,50.

(30.) Paid \$43,97 for 29 pairs of boots; what was the price per pair. Ans. \$1,51+.

(31.) If 6 yards of cloth cost \$24,48, what was the price per yard? Ans. \$4,08.

(32.) Bought 56,87 yards of cloth, at \$2,31 per yard; what was the whole cost? Ans. \$131,3697.

(33.) If a man earn \$1,001 per day, how much will he earn in 365 days? Ans. \$365,365.

(34.) What will be the cost of ,375 cord of wood at \$2 per cord? Ans. \$0,75.

(35.) What is the cost of 8,3 yards of cloth at \$5,47 per yard? Ans. \$45,401.

§ 101. *Miscellaneous Examples in Vulgar and Decimal Fractions.*

(1.) To $\frac{7}{8}$ of 74 measures of grain, were added $\frac{5}{9}$ of 49 half measures, and the whole were sold at $\frac{3}{7}$ of a dollar per measure, For how much were the whole sold ?
Ans. \$33,583+.

(2.) A kite was $\frac{1}{7}$ the lengh of its tail, the tail $\frac{3}{8}$ the length of the string, the string $9\frac{1}{3}$ times the length of the boy who held it, and the boy measured $4\frac{5}{12}$ feet. What was the length of the kite, tail, and string ? Ans. $46\frac{35}{1344}$ feet.

(3.) Triple, the $\frac{1}{2}$, and the $\frac{1}{4}$ of a certain number are equal to 104; what is that number ? Ans. $27\frac{11}{15}$.

(4.) $\frac{2}{3}$ and $\frac{3}{5}$ of a person's money amounted to \$760; how much had he ? Ans. \$600.

(5.) What sum of money is that whose 3d part, 4th part and 5th part are 94 dollars ? Ans. \$120.

(6.) $\frac{7}{8}$ of a certain number exceeds $\frac{4}{5}$ of it by 6 ; what is that number ? Ans. 80.

(7.) What number is that from which if you take $\frac{2}{7}$ of $\frac{3}{8}$, and to the remainder add $\frac{7}{16}$ of $\frac{1}{20}$, the sum will be 10 ?
Ans. $10\frac{191}{2240}$.

(8.) A father gave $\frac{7}{18}$ of his estate to one of his sons, and $\frac{7}{18}$ of the residue to another, and the surplus to his relict for life ; the difference in his son's legacies was $257\frac{1}{6}$ pounds: what was the widow's share ?
Ans. $635\frac{13}{294}$ pounds.

(9.) If to my age there added be
One half, one third, and 3 times 3,
Six score and ten their sum will be ;
What is my age ? pray show it me.
Ans. 66 years.

(10.) What is the cost of $3\frac{3}{8}$ yards of cloth, at $5\frac{3}{4}$ dollars per yard? Ans. \$19,406+.

(11.) At $5\frac{3}{4}$ dollars per yard, how much cloth can be purchased with 19,40625 dollars ? Ans. 3,375 yards.

(12.) A man died, leaving his wife in expectation of an heir, and in his will ordered, that if it were a son, $\frac{2}{3}$ of the estate should be his, and the remainder the mother's ; but if a daughter, the mother should have $\frac{2}{3}$, and the daughter $\frac{1}{3}$; but it happened she had both a son and a daughter, in consequence of which the mother's share was 2000 dollars

less than it would have been if there had been only a daughter; what would have been the mother's portion had there been only a son? Ans. $1750.

(13.) At $11,76 per hundred weight, what will $\frac{1}{8}$ of a hundred weight of sugar come to? Ans. $1,47.

(14.) How long must a laborer work, at the rate of $0,62½ per day, to earn $25? Ans. 40 days.

(15.) Traveling at the rate of $4\frac{3}{5}$ miles an hour, in how many hours will a footman go 34½ miles?

Ans. 7,5 hours.

(16.) If 85 yards of cloth be bought for 191,25 dollars, and sold at 2,87½ dollars per yard, how much is the whole profit? Ans. $53,12½.

(17.) How much butter at 9 cents a pound, will pay for 12 yards of cloth at 2,19 dollars per yard?

Ans. 292 pounds.

(18.) At 45½ dollars per acre, what is the value of $\frac{1}{5}$ of an acre of land? Ans. $9,10.

(19.) The loss of merchandize by a fire was estimated at 11372,75 dollars, $\frac{3}{5}$ of which was insured; how much was the loss after deducting the insurance?

Ans. $4549,10.

(20.) A benevolent individual whose income was 5000 dollars, devoted ,12 of it to charitable objects; how much did he give away annually? Ans. $600.

(21.) How much is $\frac{3}{4}$ of $\frac{3}{25}$ of 786½? Ans. 70,785.

(22.) A merchant buys three chests of tea, the first contains 60 and one thousandth pounds; the second, 39 and one ten thousandth pounds; the third, 26 and one tenth pounds: how much did he buy in all?

Ans. 125,1011 pounds.

(23.) B has 936 dollars, and A has 1 dollar 3 dimes and 1 mill: how much more money had B than A?

Ans. $934,699.

(24.) A merchant buys 37,5 yards of cloth at 1,25 dollar per yard: how much does the whole come to?

Ans. $46,875.

(25.) A farmer sells to a merchant 13,12 cords of wood at 4,25 dollars per cord, and 13 bushels of wheat at 1,06 dollar per bushel: he is to take in payment 13 yards of broadcloth at 4,07 per yard, and the remainder in cash: how much cash did he receive? Ans. $16,63.

PART III.

DENOMINATE FRACTIONS.

§ 102. 1. MONEY, weight, measure, &c., have each an accepted unit or whole, taken as a standard of estimation, peculiar to itself, the subdivisions of which have a constant relation to each other; the unity or single part of one subdivision being a fraction of the next above it; the denominators to which are conventional, and always understood.

2. These subdivisions vary with each separate class of quantities; and, in consequence, a different number in each makes the unit or whole.

3. The relation of these subdivisions to each other in each class of expressions is also irregular; that is, it is not from one to another by the same number or denominator. Hence, in each separate class of such expressions, the increase from right to left is by a distinct number for each place.

4. Numbers expressing quantities whose subdivisions thus form fractions with conventional denominators are called *denominate*.

5. The only difficulty in operations on such numbers consists in the attention necessary to be paid to the irregular system of subdivision, or the conventional denominators, to know how many units of each subdivision make a whole or a unit of a higher subdivision, which determines the principles of *carrying*. All the other operations on them are precisely like the same operations on simple numbers.

6. The subdivisions in the denominate quantities are so many and various, that it is necessary to have tables to recall them to memory.

Such tables we subjoin, to be thoroughly committed to memory, by the pupil, before proceeding further.

I. ENGLISH, OR STERLING MONEY.

The denominations of English or Sterling Money, are the pound, the shilling, the penny, and the farthing.

	Farthing	marked	qr.
4	Farthings make one Penny,	"	d.
12	Pence " " Shilling,	"	s.
20	Shillings " " Pound,	"	£.

		d.	qr.
	s.	1 =	4.
£.	1 =	12 =	48.
1 =	20 =	240 =	960.

This is the currency of Great Britain and her dependencies; and when the United States were Colonies of Great Britain, was their currency. More of this hereafter.

II. TROY WEIGHT.

The denominations of Troy Weight are the pound, the ounce, the pennyweight, and the grain.

	Grain	marked	gr.
24	Grains make one Pennyweight,	"	pwt.
20	Pennyweights " " Ounce,	"	oz.
12	Ounces " " Pound,	"	lb.

		pwt.	gr.
	oz.	1 =	24.
lb.	1 =	20 =	480.
1 =	12 =	240 =	5850.

By this weight are weighed gold, silver, jewels, and all liquors.

III. AVOIRDUPOIS WEIGHT.

The denominations of Avoirdupois Weight are the ton, the hundred weight, the quarter, the pound, the ounce, and the dram.

	Dram		marked	dr.
16	Drams	make one Ounce,	"	oz.
16	Ounces	" " Pound,	"	lb.
28	Pounds	" " Quarter,	"	qr.
4	Quarters	" " Hundred Weight,	"	cwt.
20	Hundred	" " Ton,	"	T.

				oz.	dr.
			lb.	1 =	16.
		qr.	1 =	16 =	256.
	cwt.	1 =	28 =	448 =	7168.
T.	1 =	4 =	112 =	1792 =	28672.
1 =	20 =	80 =	2240 =	35840 =	573440.

By this weight are weighed all things of a coarse or drossy nature, as corn, bread, butter, cheese, flesh, grocery wares, and some liquids; also all metals, except silver and gold.

In this weight the words *gross* and *nett* are used. Gross is the weight of the goods, with the boxes, casks, or bags, in which they are contained. Nett is the weight of the goods only; or what remains after deducting from the gross weight, the weight of the boxes, casks, or bags.

A hundred weight is 112 pounds, as appears from the Table. But at the present time, the merchants in our principal cities, buy and sell by the 100 pounds. For the state of New-York, the statute provides, that "the hundred weight shall consist of one hundred pounds avoirdupois, and twenty such hundreds shall constitute a ton."

		oz.	pwt.	gr.	
Note, that 1 lb. Avoirdupois	=	14	11	15½	Troy.
1 oz.	=	0	18	5½	
1 dr.	=	0	1	3½	

Hence it appears that the pound Avoirdupois contains 6999½ grains, and the pound Troy 5760; the former of which augmented by half a grain becomes 7000, and its ratio to the latter is therefore very nearly as 700 to 576, that is, as 175 to 144; consequently 144 pounds Avoirdupois are very nearly equal to 175 pounds Troy; and hence we infer that the ounce Avoirdupois is to the ounce Troy as 175 to 192.

The unit of weight is the pound, of such magnitude that a cubic foot of water at its maximum density, weighs sixty-two and a half pounds, or one thousand ounces.

IV. APOTHECARIES' WEIGHT.

The denominations of Apothecaries' Weight are the pound, the ounce, the dram, the scruple, and the grain.

					marked	
	Grain					gr.
20	Grains	make	one	Scruple,	"	℈.
3	Scruples	"	"	Dram,	"	ʒ.
8	Drams	"	"	Ounce,	"	℥.
12	Ounces	"	"	Pound,	"	℔.

			℈.	gr.
		ʒ.	1 =	20.
	℥.	1 =	3 =	60.
℔.	1 =	8 =	24 =	480.
1 =	12 =	96 =	288 =	5760.

This is the same as Troy Weight, only having some different divisions. Apothecaries make use of this weight in compounding their medicines; but they buy and sell their drugs by Avoirdupois Weight.

V. DRY MEASURE.

The denominations of Dry Measure are the chaldron, the bushel, the peck, the gallon, the quart, the pint, and the gill.

					marked	
	Gill					gi.
4	Gills	make	one	Pint,	"	pt.
2	Pints	"	"	Quart,	"	qt.
4	Quarts,	"	"	Gallon,	"	gal.
8	Quarts	"	"	Peck,	"	pk.
4	Pecks	"	"	Bushel,	"	bu.
36	Bushels	"	"	Chaldron,	"	ch.

					pt.	gi.
				qt.	1 =	4.
			gal.	1 =	2 =	8.
		pk.	1 =	4 =	8 =	32.
	bu.	1 =	2 =	8 =	16 =	64.
ch.	1 =	4 =	8 =	32 =	64 =	256.
1 =	36 =	144 =	288 =	1152 =	2304 =	9216.

By this measure are measured corn, seed, fruit, salt, and coal.

The standard gallon, dry measure, contains 268⅘ cubic inches; the bushel, 2150⅖ cubic inches; the dimensions of the measure being 8 inches deep, and 18½ inches in diameter; but the coal bushel is 19½ inches in diameter; and 36 bushels, heaped up, make a chaldron of coals, the weight of which is 3156 pounds avoirdupois.

The capacities are determined, not by measurement of the cubic contents, but by the weight of pure water, at the temperature of 62° of Fahrenheit's thermometer, contained in the vessels; the bushel holding 80, and the gallon 10 pounds avoirdupois.

VI. WINE MEASURE.

The denominations of Wine Measure are the tun, the pipe or butt, the puncheon, the hogshead, the tierce, the barrel, the gallon, the quart, the pint, and the gill.

Gill				marked	gi.
4 Gills	make	one	Pint,	"	pt.
2 Pints	"	"	Quart,	"	qt.
4 Quarts	"	"	Gallon,	"	gal.
31½ Gallons	"	"	Barrel,	"	bar.
42 Gallons	"	"	Tierce,	"	tier.
63 Gallons or 1½ tierces,	"	"	Hogshead,	"	hhd.
84 Gallons	"	"	Puncheon,	"	pun.
2 Hogsheads	"	"	Pipe,	"	P.
2 Pipes,	"	"	Tun,	"	T.

pt. gi.
qt. 1= 4.
gal. 1= 2= 8.
bar. 1 = 4= 8= 32.
tier. 1 = 31½= 126= 252=1008.
hhd. 1 =1⅓= 42 = 168= 336=1344.
pun. 1 =1½=2 = 63 = 252= 504=2016.
P. 1 =1⅓=2 =2⅔= 84 = 336= 672=2688.
T. 1=1½=2 =3 =4 =126 = 504=1008=4032.
1=2=3 =4 =6 =8 =252 =1008=2016=8064.

By this measure are measured liquors, cider, honey, and oil.

The standard gallon, wine measure, contains 231 cubic inches, and holds exactly 8lbs. of pure water at 62° of Fahrenheit.

VII. ALE, OR BEER MEASURE.

The denominations of Ale or Beer Measure are the hogshead, the barrel, the gallon, the quart, and the pint.

	Pint			marked	pt.
2	Pints	make one	Quart,	"	qt.
4	Quarts	" "	Gallon,	"	gal.
36	Gallons	" "	Barrel,	"	bar.
54	Gallons	" "	Hogshead,	"	hhd.

			qt.	pt.
		gal.	1 =	2.
	bar.	1 =	4 =	8.
hhd.	1 =	36 =	144 =	288.
1 =	$1\frac{1}{2}$ =	54 =	216 =	432.

By this measure are measured ale or beer, and milk.

Habit alone determines, in different countries where these measures are used, to which purposes the two different measures of liquids are applied besides the two liquids of which they bear the name, and these habits vary from time to time. In the State of New-York, Beer measure is little used, but the ordinary measure for all liquids is Wine measure.

The standard gallon, ale or beer measure, contains 282 cubic inches.

It is remarkable that the wine and ale gallons have the same proportion to each other as the Troy and Avoirdupois pounds have; that is, as one pound Troy is to one pound Avoirdupois, so is one wine gallon to one ale gallon.

VIII. CLOTH MEASURE.

The denominations of Cloth Measure are the French ell, the English ell, the Flemish ell, the yard, the quarter, the nail, and the inch.

	Inch			marked	in.
$2\frac{1}{4}$	Inches	make one	Nail,	"	na.
4	Nails	" "	Quarter Yard,	"	qr.
4	Quarters	" "	Yard,	"	yd.
3	Quarters	" "	Ell Flemish,	"	E. Fl.
5	Quarters	" "	Ell English,	"	E. E.
6	Quarters	" "	Ell French,	"	E. Fr.

			na.	in.
		qr.	1 =	2¼.
	yd.	1 =	4 =	9.
	1 =	4 =	16 =	36.
E. Fl.	1 =	3 =	12 =	27.
E. E.	1 =	5 =	20 =	45.
E. Fr.	1 =	6 =	24 =	54.

By this measure are measured all kinds of cloth.

IX. LONG MEASURE.

The denominations of Long Measure are the degree, the league, the mile, the furlong, the rod, the yard, the foot, the inch, and the barley corn.

Barley Corn		marked	b. c.
3 Barley Corns	make one Inch,	"	in.
12 Inches	" " Foot,	"	ft.
3 Feet	" " Yard,	"	yd.
5½ Yards or 16½ feet	" " Rod,	"	rd.
40 Rods	" " Furlong,	"	fur.
8 Furlongs	" " Mile,	"	M.
3 Miles	" " League,	"	L.

						in.	b. c.
					ft.	1=	3.
				yd.	1 =	12=	36.
			rd.	1 =	3 =	36=	108.
		fur.	1=	5½=	16½=	198=	594.
	M.	1=	40=	220 =	660 =	7920=	23760.
L.	1=	8=	320=	1760 =	5280 =	63360=	190080.
1=	3=	24=	960=	5280 =	87120 =	1045440=	3136320.

By this measure is measured length or distance.

The point, the line, the fathom, and the hand also belong to this measure. 6 points make 1 line; 12 lines make 1 inch—used in measuring the length of pendulums for clocks; the hand is 4 inches, and is used for measuring the height of horses; the fathom is 6 feet, and is used for measuring depths of water.

X. LAND, OR SQUARE MEASURE.

The denominations of Land or Square Measure are the square mile, the acre, the square rood, the square rod, the square yard, the square foot, and the square inch.

	Square Inch		marked	sq. in.
144	Square Inches make one	Square Foot,	"	sq. ft.
9	Square Feet " "	Square Yard,	"	sq. yd.
$30\frac{1}{4}$	Square Yards " "	Square Rod,	"	sq. rd.
40	Square Rods " "	Square Rood,	"	sq. r.
4	Square Roods " "	Acre,	"	A.
640	Acres " "	Square Mile,	"	M.

				sq. ft.	sq. in.
			sq. yd.	1 =	144.
		sq. rd.	1 =	9 =	1296.
	sq. r.	1 =	$30\frac{1}{4}$ =	$272\frac{1}{4}$ =	39204.
A.	1 =	40 =	1210 =	10890 =	1568160.
1 =	4 =	160 =	4840 =	43560 =	6272640.

By this measure are measured land, husbandmen and gardeners' work; also artificers' work, such as board, glass, pavements, plastering, wainscoting, tiling, flooring, and every dimension of length and breadth only.

Land is usually measured by Gunter's chain, which is 4 rods or 66 feet in length. The whole chain is divided into 100 equal parts, called links. The link is therefore $\frac{1}{25}$ part of the rod, and is $7\frac{92}{100}$ inches in length. 80 chains, or 320 rods, make 1 mile in length. 1 square chain makes 16 square rods; and 10 square chains make 1 acre.

The standard of lineal measure of the State of New-York, is the yard. Its length is by statute determined from the pendulum vibrating seconds at Columbia College, to which it bears the ratio of one million to one million eighty-six thousand one hundred and forty-one = 1 : 1,086141. The pendulum being found by the experiments of Sabine to be 39,10107 inches.

XI. SOLID MEASURE.

The denominations of Solid Measure are the cord, the ton, the solid yard, the solid foot, and the solid inch.

Solid Inch	marked	s. in.
1728 solid inches make .... 1 solid foot,		s. ft.
27 solid feet make .... 1 solid yard,		s. yd.
40 feet of round or 50 ft. of hewn timber make 1 ton,		T.
128 solid feet make .... 1 cord,		C.

A pile of wood 8 feet long, 4 feet wide, and 4 feet high, contains just one cord, since $8\times4\times4=128$.

By this measure are measured wood, timber, stone, and every substance which has the dimensions of length, breadth, and thickness.

XII. CIRCULAR MOTION.

The denominations of Circular Motion are the circumference, the sign, the degree, the minute, and the second.

	Second			marked	$''$
60	Seconds	make one	Minute,	"	$'$
60	Minutes	" "	Degree,	"	$^\circ$
30	Degrees	" "	Sign,	"	S.
12	Signs, or 360°	" "	Circumference,	"	C.

			$'$	$''$
		$^\circ$	1 =	60.
	S.	1 =	60 =	3600.
C.	1 =	30 =	1800 =	108000.
1 =	12 =	360 =	21600 =	1296000.

This measure is used in estimating latitude and longitude, and also in measuring the motions of the heavenly bodies.

XIII. TIME.

The denominations of Time are the year, the month, the week, the day, the hour, the minute, and the second.

	Second			marked	sec.
60	Seconds	make one	Minute	"	min.
60	Minutes	" "	Hour,	"	hr.
24	Hours	" "	Day,	"	da.
7	Days	" "	Week,	"	wk.
4	Weeks	" "	Month,	"	mo.

13 months, 1 day and 6 hours, or 365 days 6 hours, make one common or Julian year—yr. But 365 days, 5 hours, 48 minutes and 48 seconds, make one Solar year.

					min.	sec.
				hr.	1 =	60.
			day.	1 =	60 =	3600.
		wk.	1 =	24 =	1440 =	36400.
	mo.	1 =	7 =	168 =	10080 =	604800.
yr.	4 =	4 =	28 =	672 =	40320 =	2419200.
1 =	12 =	52 =	$365\frac{1}{4}$ =	8766 =	525960 =	31557609.

When the date of any year can be divided by 4 without a remainder, it is called leap year; then February has 29 days. The odd six hours are omitted during three years, and, on every fourth year, one day is added to February, making 366 days in the year.

A solar year contains only 12 months; the months being made up of an unequal number of days, as follows:

	No. of Days.		No. of Days.
January,	31.	July,	31.
February,	28.	August,	31.
March,	31.	September,	30.
April,	30.	October,	31.
May,	31.	November,	30.
June,	30.	December,	31.

The days in each calendar month will be more easily remembered, by committing to memory the following lines:

Thirty days hath September,
April, June, and November,
All the rest have thirty-one,
Except the second month alone,
Which hath but twenty-eight in fine,
Till leap year gives it twenty-nine.

XIV. MISCELLANEOUS.

12 things	make	one	dozen, marked	doz.
12 dozen	"	"	gross, "	gs.
12 gross	"	"	great gross.	
20 things	"	"	score.	
24 sheets of paper	"	"	quire.	
20 quires "	"	"	ream.	
10 quires "	"	"	token.	

56lb of corn	make one	bushel.
56lb of rye	" "	"
60lb of wheat	" "	bushel.
4½ bushels of wheat	" "	barrel of flour.
196lb	" "	" " "
200lb	" "	" " pork.
200lb	" "	" " beef.
200lb	" "	" " shad.
300lb	" "	tierce of salmon.
112lb	" "	quintal of fish.
14lb	" "	stone of iron or wood.
8lb	" "	stone of meat.

BOOKS.

A sheet folded in	2	leaves, is called	a folio.
"	4	"	a quarto, or 4to.
"	8	"	an octavo, or 8vo.
"	12	"	a duodecimo, or 12mo.
"	18	"	an 18mo.

§ 103. For combination, and other purposes oftentimes, it is expedient to change the different denominations in any one kind of quantity to one denomination; as pounds, shillings and pence, to pence; and upon this, operations may be performed as on simple or abstract numbers. Hence we have,

§ 104. 1. Case I. *To change higher denominations to lower.*

Rule. I. Multiply the highest denomination given by the number of the next lower denomination which makes one of that higher; and add to the product the next lower denomination in the given number.

II. Multiply this sum by the number of the next lower denomination, which makes one of that higher, and add to the product the next lower denomination in the given number.

III. Proceed in this manner through all the denominations to the lowest given, and the number last obtained will be the result required.

2. *Illustration.* Change 1cwt. 3qrs. 13lbs. 7oz. 3dr. to drams.

cwt.	qrs.	lbs.	oz.	dr.
1	3	13	7	3
4				
—				
7				
28				
—				
69				
14				
—				
209×16				
1261				
—				
3351×16				
20106				
—				
53619				

(1.) Here we multiply the highest denomination, 1cwt., by 4, the number of the next lower denomination, quarters, which makes one of that higher, and to the product we add the 3 of the next lower denomination in the given number.

(2.) We then multiply this sum by 28, the number of the next lower denomination which makes one of that higher, and to the product add the 13, the next lower denomination of the given number.

(3.) Then we multiply by 16, the number of the next lower denomination which makes one of that higher, and to the product add the 7, the next lower denomination of the given number.

(4.) We then again multiply by 16, the number of the next lower denomination which makes one of that higher, and to the product add the 3 of the lowest denomination in the given number, and the sum 53619 is the result required.

3. *Explanation.* It has been stated that each of the parts in any subdivision of denominate fractions may be regarded as a unit in that subdivision (§102. 1). This ope-

ration then, is simply that of changing a certain number of such units to a lower subdivision, or changing a whole number to a fraction of a specified denominator (§ 55, 1).

4. *Examples.*

(1.) Change £46 5s. 11d. 3qr. to farthings. Ans. 44447qr.

(2.) Change £76 to pence. Ans. 18240d.

(3.) Change £23 to farthings. Ans. 22080qr.

(4.) Change 62lb. 7oz. 14pwt. 18gr. to grains. Ans. 360834gr.

(5.) Change 23lb. 9oz. 6pwt. 22gr. to grains. Ans. 136966gr.

(6.) Change 59lb. 13pwt. 5gr. to grains. Ans. 340157gr.

(7.) Change 4cwt. 3qr. 26lb. 10oz. 12dr. to drams. Ans. 143020dr.

(8.) Change 12 tons to ounces. Ans. 430080oz.

(9.) Change 1℔. 1℥. 1ʒ. 1℈. 1gr. to grains. Ans. 6321gr.

(10.) Change 30yd. 3qr. 3na. to nails. Ans. 495na.

(11.) Change 6 pipes 1hhd. 1gi. to gills. Ans. 26209gi.

(12.) Change 16 barrels 21 gallons to quarts. Ans. 2100qt.

(13.) Change 47 barrels of beer to pints. Ans. 13536pt.

(14.) Change 36 gallons of beer to pints. Ans. 288pt.

(15.) Change 75 bu. of corn to pints. Ans. 4800pt.

(16.) Change 6ch. 9bu. 3pk. to gills. Ans. 57792gi.

(17.) Change 42ch. to pecks. Ans. 6048pk.

(18.) Change 8l. 2m. 6fur. 16rd. 3y. 2ft. 9in. 2b.c. to barley corns. Ans. 5094569b.c.

(19.) Change 7fur. 36rd. 9ft. to inches. Ans. 62676in.

(20.) Change 3 square miles to square rods. Ans. 307200sq. rd.

(21.) Change one acre to square inches. Ans. 6272640sq. in.

(22.) Change 72 tons of hewn timber to cubic inches. Ans. 6220800c. in.

(23.) Change 88 tons 20ft. of round timber to cubic inches. Ans. 6117120c. in.

(24.) Change 25 cords of wood to inches.
Ans. 5529600in.

(25.) Change 6 signs, 21 degrees, 40 minutes to seconds.
Ans. 726000″.

(26.) Change 1 circumference, 6 signs, 25 degrees to minutes. Ans. 33900′.

(27.) Change 49 weeks to seconds.
Ans. 29635200sec.

(28.) Change 364 days, 5 hours 48 minutes 48 seconds to seconds. Ans. 31470528 sec.

(29.) Change 2 C. to seconds. Ans. 2592000″.

(30.) Change 1 C. 5 s. 28° 15′ to minutes.
Ans. 33295′.

(31.) Change 5 chaldrons 32 bushels to pecks.
Ans. 848 pks.

(32.) Change 59 miles 7 furlongs 38 rods to rods.
Ans. 19198 rd.

(33.) Change 4℔. 2℥. 1℈. 13 grains to grains.
Ans. 24033 gr.

(34.) Change 28 E. Fl. 1 qr. to nails. Ans. 340 na.

5. If the lower expression is wanted in the form of a vulgar fraction, we have only to write under it the conventional denominator of that lower denomination; as 48 qrs.$=\frac{48}{4}$. £1 3s. 4d. 3qrs.=1123 qrs.$=\frac{1123}{4}$.

6. If the denominations to be changed be in the form of vulgar fractions, the multiplications are performed on the numerators. The fractions are thus successively changed from higher to lower denominations; each fraction of a lower denomination being added to the higher when changed to that lower denomination, till a vulgar fraction is obtained expressive of the quantity in the lower denomination sought.

7. *Illustrations.* (1.) Change £$\frac{2}{9}$ to the fraction of a penny.

$$\tfrac{2}{9}\times20\times12=\tfrac{480}{9}=\tfrac{160}{3}.$$

(2.) Change £$\frac{3}{4}$ $\frac{5}{6}$s. $\frac{3}{5}$d. to the fraction of a penny.

$$£\tfrac{3}{4}=\tfrac{3\times20}{4}\text{s.}=\tfrac{60}{4}\text{s.}$$

Then, $£\frac{3}{4}+\frac{5}{6}\text{s.}=\frac{60}{4}\text{s.}+\frac{5}{6}\text{s.}=\frac{380}{24}\text{s.}=\frac{380\times12}{24}\text{d.}=\frac{4560}{24}\text{d.}$

Then, $£\frac{3}{4}+\frac{5}{6}\text{s.}+\frac{3}{5}\text{d.}=\frac{4560}{24}\text{d.}+\frac{3}{5}\text{d.}=\frac{23872}{120}\text{d.}=\frac{2984}{15}\text{d.}$

(1.) Here, in the first illustration, we multiply the numerator by 20 to change it to the fraction of a shilling, and that product by 12 to change it to the fraction of a penny; and we have $\frac{480}{9}$d. $=\frac{160}{3}$d. the required fraction.

(2.) In the second illustration, we first multiply the numerator of £$\frac{3}{4}$ by 20 to change it to the fraction of a shilling and we obtain $\frac{60}{4}$s.; we then add $\frac{5}{6}$s. to this result which gives $\frac{380}{24}$s.

(3.) We then multiply the numerator of $\frac{380}{24}$s. by 12, to change it to the fraction of a penny, and we obtain $\frac{4560}{24}$d.; then we add $\frac{3}{5}$d. to this result which gives $\frac{23872}{120}$d. $=\frac{2984}{15}$d., the fraction required.

8. *Explanation.* When the numerator denotes a part or parts of a higher denomination, it is obvious that it must be equivalent to a greater number of a lower denomination. This greater number of a lower denomination it is made to denote then, by being multiplied; as £$\frac{2}{9}$ is equal to $\frac{40}{9}$s. $=\frac{480}{9}$d.

9. *Examples.*

(1.) Change $\frac{4}{5}$ of a pound to the fraction of a penny.
Ans. $\frac{960}{5}$d.

(2.) Change $\frac{3}{13}$ of a month to the fraction of a day.
Ans. $\frac{84}{13}$da.

(3.) Change $\frac{5}{171}$ of a bushel to the fraction of a quart.
Ans. $\frac{160}{171}$qt.

(4.) Change $\frac{1}{20}$ of $\frac{3}{5}$ of a cwt. to the fraction of a pound.
Ans. $\frac{336}{100}$lb.

(5.) Change $\frac{3}{1460}$ of a year to the fraction of a day.
Ans. $\frac{1095}{1460}$d.

(6.) Change $\frac{1}{108}$ of a lb. troy to the fraction of a pwt.
Ans. $\frac{20}{9}$pwt.

(7.) Change $\frac{1}{960000}$ of a mile to the fraction of a barley corn.
Ans. $\frac{99}{500}$b. c.

(8.) Change $\frac{3}{160}$ of an ell English to the fraction of a nail.
Ans. $\frac{3}{8}$na.

(9.) Change $\frac{1}{9490}$ of a year to the fraction of an hour.
Ans. $\frac{12}{13}$hr.

(10.) Change $\frac{1}{28}$ of a pound aviordupois to the fraction of an oz.
Ans. $\frac{4}{7}$oz.

§ 105. It is also very often required to change one denomination in one of the kinds of quantity to different denominations of the same, when the denomination given is lower than the highest in that particular class or kind of quantity; as pence, to shillings and pounds. Hence we have,

§ 106. 1. CASE II, *To change lower denominations to higher.*

RULE. I. Divide the given number by its conventional denominator, that is, by the number of that denomination which makes one of the next higher, and write the remainder, (if any,) which will be of the denomination of the dividend, at the right.

II. Divide this quotient by the number of this denomination which makes one of the next higher; and write the remainder, (if any,) as before.

III. Proceed in the same manner through all the denominations to that which is required; and the last qoutient with the several remainders, (if any,) taken in order from the last, will be the required result.

2. *Illustration.* Change 9563 lbs. to cwt. &c.

```
28)9563
   -----
  4)341  15
    ---
     85  1  15
```

(1.) Here we first divide the given number by its conventional denominator, that is, by the number of parts which it takes of that denomination to make one of the next higher; and obtain as a quotient 341 qr. and a remainder of 15 pounds which we write at the right.

(2.) We then divide this first quotient by its conventional denominator, that is, by the number of parts of this denomination which makes one the next higher and obtain as a quotient 85 cwt. and a remainder of 1 qr.

(3.) We then write the first emainder at the right of the last quotient and remainder, for the result required.

3. *Explanation.* This case is precisely the reverse of the

last. The reason of the Rule then is obvious. As by multiplying successively the several denominations by the number of parts in the next lower denomination which makes one of the higher, we obtain them in their lowest denomination; so by reversing the operation, by dividing by the same numbers, in order from lowest to highest, we obtain successivly the higher denominations sought.

4. *Examples.*

(1.) Change 7195 pence to pounds, &c.
Ans. £29 19s. 7d.

(2.) Change 68795qr. to pounds, shillings, &c.
Ans. £71 13s. 2d. 3qr.

(3.) Change 11519 grains to pounds, &c.
Ans. 1lb. 11oz. 19pwt. 23gr.

(4.) Change 5835pwt. to pounds, &c.
Ans. 24lbs. 3oz. 15pwt.

(5.) Change 480 drams to pounds, &c.
Ans. 1lb. 14oz.

(6.) Change 430080 ounces to tons. Ans. 12 tons.

(7.) Change 6348 scruples to pounds, &c.
Ans. 22lbs. 4℥.

(8.) Change 50272 grains to pounds, &c.
Ans. 8℔ 8℥ 5ʒ 2℈ 12gr.

(9.) Change 35932 nails to ells English, &c.
Ans. 1796 E. E. 3 qrs.

(10.) Change 9173 nails to yards, &c.
Ans. 573 yds. 1 qr. 1 na.

(11.) Change 68160 rods to miles. Ans. 213 miles.

(12.) Change 190080 inches to leagues.
Ans. 1 league.

(13.) Change 8173440 b. c. to miles. Ans. 43 miles.

(14.) Change 31536000 seconds to years.
Ans. 1 year.

(15.) Change 175320 hours to J. years. Ans. 20 years.

(16.) Change 89763 square yards to acres, &c.
Ans. 18 A. 2 sq. r. 7 sq. rd. 11 sq. yd. 2 sq. ft. 36 sq. in.

(17.) Change 1296000 solid inches to tons of hewn timber. Ans. 15 tons.

(18.) Change 5667840 solid inches to tons of round timber. Ans. 82 tons.

(19.) Change 5529600 solid inches to cords.
Ans. 25 cords.

(20.) Change 1020300 seconds to signs, &c.
Ans. 9 signs 13° 25′.

(21.) Change 9758 pints to pipes, &c.
Ans. 9p. 1hhd. 22gal. 3qts.

(22.) Change 1008qt. to tuns. Ans. 1 tun.

(23.) Change 13680pt. of beer to barrels, &c.
Ans. 47bar. 18gal.

(24.) Change 9376qt. to bushels. Ans. 293bu.

(25.) Change 5376pk. to ch., &c. Ans. 37ch. 12bu.

§ 107. When we have occasion to make use of a number consisting of several denominations as a simple or abstract number, instead of changing the several parts to the lowest denomination contained in it, as is done in Case I., we may often find it expedient to change the lower denominations to a fraction of a higher. Hence we have,

§ 108. 1. CASE III. *To change a denominate to a vulgar fraction of a higher denomination.*

RULE. I. Write under the lowest of the given denominations its conventional denominator, that is, the number of that denomination which makes one of the next higher; and the fraction thus obtained will express a part or parts of the next higher denomination.

II. Annex this fraction to the next higher of the given denominations, and change the expression thus made to an improper fraction.

Then divide this fraction, by multiplying its denominator, by the number of that denomination which makes one of the next higher; the fraction thus obtained will express a part or parts of the next higher denomination.

III. Annex this fraction to the next higher of the given denominations; change the expression thus made to an improper fraction, then divide as before.

IV. Proceed in this manner till a vulgar fraction is obtained which expresses the quantity in the denomination sought.

2. *Illustrations.*

(1.) Change 3s. to the fraction of a pound.

$$3s. = £\tfrac{3}{20}.$$

Here we write 20, the number of shillings which makes a pound, or the conventional denominator of the shillings, under the given number 3s., and we have the result in parts of the next higher denomination.

(2.) Change £4 15s. 9d. to the fraction of a pound.

$$9d. = \tfrac{9}{12}s.$$

Then, £4 15s. 9d. $= £4\ 15\tfrac{9}{12}s.$ $\quad 15\tfrac{9}{12}s. = \tfrac{189}{12}s. = £\tfrac{189}{12 \times 20} = £\tfrac{189}{240}.$ $= \frac{63}{80}$

Then, £4 15s. 9d. $= £4\tfrac{189}{240} = £\tfrac{1149}{240}.$ $= \frac{383}{80}$

(1.) Here under the lowest denomination, the pence, we write 12, the number of pence which makes a shilling, and we have a fraction as the result, $\tfrac{9}{12}$ of a shilling, which we annex to the shillings, the next higher denomination.

(2.) Then we change the expression $15\tfrac{9}{12}$s., to an improper fraction $\tfrac{189}{12}$s., which we divide by multiplying its denominator by 20, the number of shillings which makes a pound, and we have a fraction as the result, $\tfrac{189}{240}$ of a pound.

(3.) We annex this fraction to the pounds, the next higher denomination, and have the expression £$4\tfrac{189}{240}$, which, changed to an improper fraction, gives £$\tfrac{1149}{240}$, the required fraction.

Again. (3.) Change 2wk. 1da. 6hr. to the fraction of a month.

$$2w.\ 1d.\ 6h. = 2w.\ 1\tfrac{6}{24}d. = 2w.\ \tfrac{30}{24}d. = 2\tfrac{30}{168}w. = \tfrac{366}{168}w. = \tfrac{366}{672}\text{ month.}$$

(1.) Here under the lowest denomination, we write 24, the number of hours in a day, and we have a fraction as the result, $\tfrac{6}{24}$ of a day, which we annex to the next higher denomination.

(2.) We then change the expression $1\tfrac{6}{24}$d. to an improper fraction $\tfrac{30}{24}$d., which we divide, by multiplying its denominator by 7, the number of days in a week, and have $\tfrac{30}{168}$w. as the result.

(3.) This fraction we annex to the weeks, the next higher denomination, and have the expression $2\frac{30}{168}$w., which, changed to an improper fraction, gives $\frac{366}{168}$w. By dividing this by 4, the number of weeks in a month, we have $\frac{366}{672}$ of a month, the fraction required.

3. *Explanation.* Writing under a denominate fraction its conventional denominator, indicates its division by that denominator, and thus changes the denomination, in effect, to the next higher. The principles by which lower denominations are changed to higher by such a division executed, are fully developed under Case II. The only difference between that case and this, is that here is indicated what there is performed.

4. *Examples.*

(1.) Change 6 qts. to the fraction of a bushel. Ans. $\frac{3}{16}$bu.

(2.) Change 12 oz. to the fraction of a pound avoirdupois. Ans. $\frac{3}{4}$lb.

(3.) Change 3 qr. 2 na. to the fraction of an ell English. Ans. $\frac{7}{16}$E. E.

(4.) Change 12 hr. 30 min. to the fraction of a day. Ans. $\frac{25}{48}$da.

(5.) Change 2ʒ 2℈ to the fraction of an ounce. Ans. $\frac{1}{3}$℥.

(6.) Change 2 qr. 24 lb. to the fraction of a cwt. Ans. $\frac{5}{7}$cwt.

(7.) Change 6 fur. 26 po. 3 yds. 2 ft. to the fraction of a mile. Ans. $\frac{5}{6}$m.

(8.) Change 9 gal. to the fraction of a hogshead. Ans. $\frac{1}{7}$hhd.

(9.) Change 1 inch to the fraction of a mile. Ans. $\frac{1}{63360}$m.

(10.) Change 3 qt. 1 pt. to the fraction of hogshead. Ans. $\frac{1}{72}$hhd.

(11.) Change 3 hr. 3 m. to the fraction of a day. Ans. $\frac{183}{1440}$da.

(12.) Change 237 da. 19 hr. 45 m. $52\frac{16}{17}$ sec. to the fraction of a Julian year. Ans. $\frac{12}{17}$ J. yr.

(13.) Change 3 sq. r. $17\frac{1}{4}$ sq. rd. to the fraction of an acre. Ans. $\frac{4}{7}$A.

(14.) Change 4 cwt. 2 qr. 12lb. 14 oz. $12\frac{4}{13}$ dr. to the fraction of a ton. Ans. $\frac{3}{13}$ T.

(15.) Change 4 qr. $1\frac{1}{2}$ na. to the fraction of an ell English. Ans. $\frac{7}{8}$ E. E.

(16) Change 10 oz. 10 pwt. 10 gr. to the fraction of a pound. Ans. $\frac{505}{576}$lb.

(17.) Change 14° 34′ 42″ to the fraction of a sign. Ans. $\frac{8747}{18000}$ S.

(18.) Change 14 s. ft. 1444 s. in. to the fraction of a solid yard. Ans. $\frac{6409}{11664}$ s. yd.

5. It will be seen at once from the above (Rule. II. second paragraph) that if the denominations to be changed be in the form of vulgar fractions, the operation of multiplication alone has to be performed on the denominators.

6. The fractions are thus successively changed from lower to higher denominations; each fraction of a higher denomination being added to the lower when changed to that higher, till a vulgar fraction is obtained expressive of the quantity in the higher denomination sought.*

7. *Illustration.*

Change $\frac{4}{5}$ of a penny to the fraction of a pound.

$$\frac{4}{5\times 12\times 20}=£\frac{4}{1200}=£\frac{1}{300}.$$

Here we first multiply the denominator of $\frac{4}{5}$d. by 12, to change it to the fraction of a shilling, and then that result by 20, to change it to the fraction of a pound, and we obtain $£\frac{4}{1200}=£\frac{1}{300}$, the required fraction.

8. *Explanation.* Multiplying the denominator divides the fraction (§ 53. 5 (4.)); and by division lower denominations are changed to higher (§ 106. 1).

*It will often be found more convenient to reduce the several parts of the compound number to the lowest denomination, as by the preceding article (§ 104. 1), for a numerator, and to take for the denominator so many of this denomination as it takes to make one of that to which the expression is to be reduced; thus, £4 15s. 9d. being 1149d., is equal to $£\frac{1149}{240}$, because 1d. is $£\frac{1}{240}$.

Let the pupil be required to solve the examples which precede, in both ways.

9. *Examples.*

(1.) Change $\frac{6}{7}$ of a pound to the fraction of a cwt.
Ans. $\frac{3}{392}$cwt.

(2.) Change $\frac{2}{3}$ of a farthing to the fraction of a pound.
Ans. £$\frac{1}{1440}$.

(3.) Change $\frac{4}{5}$ of a pwt. to the fraction of a pound.
Ans. $\frac{1}{300}$lb.

(4.) Change $\frac{2}{3}$ of a quart to the fraction of a bushel.
Ans. $\frac{1}{48}$bu.

(5.) Change $\frac{4}{7}$ of an inch to the fraction of a yard.
Ans. $\frac{1}{63}$yd.

(6.) Change $\frac{4}{7}$ of an ounce to the fraction of a pound avoirdupois.
Ans. $\frac{1}{28}$lb.

(7.) Change $\frac{6}{7}$ of a gallon to the fraction of a hogshead.
Ans. $\frac{2}{147}$hhd.

(8.) Change $\frac{16}{17}$ of a minute to the fraction of a day.
Ans. $\frac{1}{1530}$da.

(9.) Change $\frac{3}{8}$ of a nail to the fraction of an ell English.
Ans. $\frac{3}{160}$ E. E.

(10.) Change $\frac{12}{13}$ of an hour to the fraction of a year of 365 days.
Ans. $\frac{1}{9490}$yr.

(11.) Change $\frac{1}{15}$ of a grain to the fraction of a dram.
Ans. $\frac{1}{900}$ʒ.

(12.) Change $\frac{1}{75}$ of a dram to the fraction of a quarter.
Ans. $\frac{1}{537600}$qr.

(13.) Change $\frac{8}{21}$ of a minute to the fraction of a sign.
Ans. $\frac{1}{4725}$ S.

§ 109. 1. Case IV. *To change a denominate fraction to the decimal of a higher.*

Rule. I. Divide the lowest of the given denominations by its conventional denominator, that is, by the number of that denomination which makes one of the next higher; and annex the quotient, which expresses the result in a decimal of the next higher denomination, to that higher denomination.

II. Divide the next higher denomination, with the quotient last obtained annexed, by the number of that denomination which makes one of the next higher, and so on as before.

III. Proceed in this manner till the quantity is expressed in a decimal of the denomination sought.

2. *Illustrations.*

(1.) Change 15 shillings to the decimal of a pound.

$$15s.=£\tfrac{15}{20}=£0,75.$$

Here we divide the given denomination by its conventional denominator, 20, the number of shillings which makes a pound, as in Case I. of decimals.

(2.) Change 12s. 6d. 3qr. to the decimal of a pound.

$$\begin{array}{rl} 4)3, & \text{qr.} \\ \hline 12)6,75 & \text{d.} \\ \hline 20)12,5625 & \text{s.} \\ \hline £0,628125 & \end{array}$$

(3.) Here we first divide the lowest denomination given, farthings, by 4, its conventional denominator, and annex the result ,75d. to the next higher denomination, which gives 6,75d.

(4.) We then divide this result, 6,75d., by 12, the number of pence in a shilling, and obtain ,5625s., which we annex to the next higher denomination, which gives 12,5625s.

(5.) This result we divide by 20, the number of shillings in a pound, and obtain £0,628125, the required decimal.

3. *Explanation.* The character of denominate fractions as subdivisions of unity (§ 102, 1), and the explanation to Case I. of decimals sufficiently explain this operation. For, the divisions are of the numerators by their implied denominators, on the principle of changing a vulgar to a decimal fraction.

4. *Examples.*

(1.) Change 3qr. 16lb. to the decimal of a cwt.
Ans. ,8928571+cwt.

(2.) Change 2 sq. r. and 20 sq. rd. to the decimal of an acre.
Ans. ,625 A.

(3.) Change 2cwt. 3qr. 24lb. to the decimal of a ton.
Ans. ,14821428+T.

(4.) Change 3 furlongs 16 rods to the decimal of a mile.
Ans. ,425m.

(5.) Change 9s. 8d. to the decimal of a pound.
Ans. £0,4833+.

(6.) Change 3qr. 2na. to the decimal of a yard.
Ans. ,875yd.

(7.) Change 55m. 37sec. to the decimal of an hour.
Ans. ,926944+hr.

(8.) Change 5fur. 12rd. to the decimal of a mile.
Ans. ,6625m.

(9.) Change 1gal. to the decimal of a hogshead.
Ans. ,0158730+hhd.

(10.) Change 2 sq. r. 16 sq. rd. to the decimal of an acre.
Ans. ,6 A.

(11.) Change 2ft. 6in. to the decimal of a yard.
Ans. ,833333+yd.

(12.) Change 7oz. 19pwt. to the decimal of a pound.
Ans. ,6625 lb.

(13.) Change 4½ calendar months to the decimal of a year.
Ans. ,375yr.

(14.) Change 15s. 9d. 3qr. to the decimal of a pound.
Ans. £0,790625.

§ 110. 1. CASE V. *To change a vulgar to a denominate fraction.*

RULE. I. Multiply the numerator by the number of the next lower denomination which makes one of that higher; divide by the denominator, and the quotient will be the result in that lower denomination.

II. If there be a remainder, after the first division, multiply it by the number of the next lower denomination which makes one of that higher, and divide by the denominator to the fraction as before.

III. Proceed in the same manner till there is no remainder, or till a quotient is obtained in the lowest denomination of the given quantity.

2. *Illustration.*

Change $\frac{5}{7}$ of a pound to a denominate fraction.

$$£\frac{5}{7}=\frac{5\times20}{7}\text{s.}=\frac{100}{7}\text{s.}=14\text{s.}+\frac{2}{7}\text{s.}$$

$$\frac{2}{7}\text{s.}=\frac{2\times12}{7}\text{d.}=\frac{24}{7}\text{d.}=3\text{d.}+\frac{3}{7}\text{d.}$$

$$\frac{3}{7}\text{d.}=\frac{3\times4}{7}\text{qr.}=\frac{12}{7}\text{qr.}=1\frac{5}{7}\text{qr.};\text{ therefore, }£\frac{5}{7}=14\text{s. }3\text{d. }1\tfrac{5}{7}\text{qr.}$$

3. *Explanation.* Changing a vulgar to a denominate fraction is, in effect, changing a compound fraction to a single one. For the vulgar fraction is invariably a part of the denominate fraction to which it is to be changed. The parts of the denomination to which it is required to change a vulgar fraction which are necessary to make unity of that denomination, form the numerator of the second fraction of which the first is a part. The multiplication, therefore, of the numerator of the vulgar fraction by the number of parts which it takes of the next lower denomination to make one of that higher, is really the multiplication of the numerators of a compound fraction; and the denominator to the second being unity, the retention of the denominator of the first is evidently the same as the multiplication of that denominator by unity; it, therefore, is the the denominator of the single fraction.

The same reasoning holds in each step to the lowest denomination.

4. *Examples.*

(1.) Change £$\frac{7}{40}$ to a denominate fraction.
Ans. 3s. 6d.

(2.) Change $\frac{17}{29}$ cwt. to a denominate fraction.
Ans. 2 qrs. 9lb. 10 oz. $7\frac{21}{29}$ dr.

(3.) Change $\frac{6}{7}$ of an acre to a denominate fraction.
Ans. 3 sq. r. $17\frac{1}{7}$ sq. rd.

(4.) Change $\frac{3}{5}$ of a lb. troy to a denominate fraction.
Ans. 7 oz. 4 pwt.

(5.) Change $\frac{5}{6}$ of a mile to a denominate fraction.
Ans. 6 fur. 26 rd. 3 yd. 2 ft.

(6.) Change $\frac{2}{3}$ yd. to a denominate fraction.
Ans. 2 qrs. $2\frac{2}{3}$ na.

(7.) Change $\frac{9}{24}$ of a shilling to a denominate fraction.
Ans. 4d. 2 qr.

(8.) Change $\frac{7}{8}$ of an ell English to a denominate fraction.
Ans 4 qrs. $1\frac{1}{2}$ na.

(9.) Change $\frac{4}{9}$ of a sign to a denominate fraction.
Ans. 13° 20′.

(10.) Change $\frac{4}{9}$ of a month to a denominate fraction.
Ans. 1wk. 5da. 10hr. 40min.

(11.) Change $\frac{9}{12}$ of mile to a denominate fraction.
Ans. 6 fur.

(12.) Change $\frac{3}{15}$ of a scruple to a denominate fraction.
Ans. 4 gr.

(13.) Change $\frac{9}{14}$ of a hogshead to a denominate fraction.
Ans. 40gal. 2qt.

(14.) Change $\frac{8}{9}$ of a cwt. to a denominate fraction.
Ans. 3qr. 15lb. 8oz. $14\frac{2}{9}$dr.

(15.) Change $\frac{4}{5}$ of an acre to a denominate fraction.
Ans. 3sq. r. 8sq. rd.

(16.) Change $\frac{1}{8}$ of a solid yard to a denominate fraction.
Ans. 3 s. ft. 648 s. in.

(17.) Change $\frac{2}{3}$ of a qr. to a denominate fraction.
Ans. 18lb. 10oz. $10\frac{2}{3}$dr.

(18.) Change $\frac{5}{6}$ of a degree to a denominate fraction.
Ans. 50′.

§ 114. 1. CASE VI. *To change a decimal to a denominate fraction.*

RULE. I. Multiply the decimal by the number of the next lower denomination which makes one of that higher; write the point in the product at the left of as many figures as there are decimals in the given number.

II. Multiply the decimal then remaining by the number of the next lower denomination which makes one of that higher, and write the point as before.

III. Proceed in this manner through all the denominations, to the lowest.

IV. The several denominations at the left of the points, taken in order from the first, are the denominate fractions required.

2. *Illustration.*

Change ,617 cwt. to a denominate fraction.

```
 Cwt.
 ,617
    4
 -----
 2,468
    28
 -----
 3744
 936
 ---------
 13,104×16
   624
 ---------
 1,664×16
   3984
 ---------
 10,624
```

(1.) Here we first multiply the decimal by the 4, the number of parts of the next lower denomination which makes one of that higher, and write the point in the product as in the multiplication of decimals; at the left of as many figures as there are decimals in both factors (§ 95. Rule. III.).

(2.) We then multiply the decimal remaining by 28, for the same reason as before, and write the point by the same rule.

Thus we proceed to the lowest denomination, till we have multiplied by the drams.

(3.) We then have at the left of the points, as the denominate fraction required, 2qr. 13lb. 1oz. 10,624dr.

3. *Explanation.* This operation is directly the reverse of that in Case IV. The reason of the Rule then is obvious.

As, by dividing by the conventional denominator of the given numbers, we obtain their decimal; so, by reversing the operation and multiplying the decimal, must we obtain its equivalent denominate numbers.

4. *Examples.*

(1.) Change £0,75 to a denominate fraction.
Ans. 15s.

(2.) Change ,9 of an acre to a denominate fraction.
Ans. 3 sq. r. 24 sq. rd.

(3.) Change ,66 of a yd. to a denominate fraction.
Ans. 2qr. 2na. 1,26in.

(4.) Change ,376 of a month to a denominate fraction.
Ans. 1wk. 3da. 12hr. 40min. 19,2sec.

(5.) Change ,676 of a cwt. to a denominate fraction.
Ans. 2qr. 19lb. 11oz.+.

(6.) Change ,875 of a hhd. of wine to a denominate fraction.
Ans. 55gal. 1pt.

(7.) Change ,8593 of a lb. troy to a denominate fraction.
Ans. 10oz. 6pwt. 5,5gr.

(8.) Change ,713 of a day to a denominate fraction.
Ans. 17hr. 6min. 4,32sec.

(9.) Change ,002084lb. troy to a denominate fraction.
Ans. 12,003840gr.

(10.) Change ,625cwt. to a denominate fraction.
Ans. 2qr. 14lb.

(11.) Change ,625gal. to a denominate fraction.
Ans. 2qt. 1pt.

(12.) Change ,3375T. to a denominate fraction.
Ans. 6cwt. 3qr.

(13.) Change ,05A. to a denominate fraction.
Ans. 8sq. rd.

(14.) Change ,875P. to a denominate fraction.
Ans. 1hhd. 47gal. 1qt.

(15.) Change ,125hhd. of beer to a denominate fraction.
Ans. 6gal. 3qt.

(16.) Change ,089 mile to a denominate fraction.
Ans. 28rd. 7ft. 11,04in.

(17.) Change ,09375lb. avoirdupois to a denominate fraction.
Ans. 1oz. 8dr.

(18.) Change ,86cwt. to a denominate fraction.
Ans. 3qr. 12lb. 5oz. 1,92dr.

(19.) Change ,07 of 32gal. of wine to a denominate fraction.
Ans. 2gal. 1,92pt.

(20.) Change ,575 C. to a denominate fraction.
Ans. 6s. 2°. 42′.

§ 112. 1. CASE VII. *To add denominate fractions.*

RULE. I. Write the numbers of the same denomination under each other.

II. Add together the right or lowest column; divide the sum by the number of that denomination which makes one of the next higher; write down the remainder, and retain the quotient, to be added to the next higher column.

III. Add to the next higher column the number thus retained; divide the sum by the number of that denomination, which makes one of the next higher; write down the remainder, &c. as before.

IV. Proceed in this manner to the last column, which add as simple or abstract numbers.

2. *Illustration.* Add together £2 8s. 7d. 3qr.+£6 4s. 3d. 1qr.+£1 16s. 10d. 2qr.

£.	s.	d.	qr.
2	8	7	3
6	4	3	1
1	16	10	2
£10	9	9	2

(1.) Here we write the numbers of the same denomination under each other, farthings under farthings, pence under pence, and so on.

(2.) We then add the first column at the right, that of farthings. The sum of it we find to be 6. This contains a unit of the next higher denomination and two over, or one penny and two farthings. We write the 2qr. under the column of farthings and retain the 1d. to be added to the column of pence.

(3.) We next add the column of pence, the sum of which, with the 1d. retained, we find to be 21. In 21 pence, there are 1s. and 9d. We write the 9d., under the column of pence, and retain the 1s. to be added to the next column.

(4.) In this manner we proceed till we come to the last or left hand column, which, being the highest denomination, we add as simple numbers, and the whole sum is £10. 9s. 9d. 2qr.

3. *Explanation.* The peculiarity in this addition consists in the principle of carrying by the conventional denominator to the parts added, which being arbitrary, has to be learned from the tables, and forms the only difficulty in the operation (§ 102. 5).

4. *Examples.*

(1.) Add £92 19s. 5d. 2qr.+£5 14s. 8d. 1qr.+£22 2s. 11d.+£98 4s. 5d. together. Ans. £219 1s. 5d. 3qr.

(2.) Add 56 A. 3 sq. r. 37 sq. rd.+59 A. 2 sq. r. 28 sq. rd.+75 A. 1 sq. r. 18 sq. rd. together.
Ans. 192 A. 0 sq. r. 3 sq. rd.

(3.) Bought 3 hogsheads of sugar, weighing as follows: 9cwt. 2qr. 18lb.; 8cwt. 3qr. 12lb.; 7cwt. 2qr. 19lb.; what did they all weigh? Ans. 26cwt. 0qr. 21lb.

(4.) Admit a man traveled in one day 27 M. 2 fur.; in another, 32 M. 7 fur. 33 rd.; in another, 19 M. 7 fur. 16 rd.; and in another, 12 M. 5 fur.; how far did he travel in all?
Ans. 92 M. 6 fur. 9 rd.

(5.) A landlord has four farms; the first contains 120 A. 3 sq. r.; the second, 150 A.; the third, 215 A. 1 sq. r.; and the fourth, 96 A. 2 sq. r. 20 sq. rd.; how many acres are there in all? Ans. 582 A. 2 sq. r. 20 sq. rd.

(6.) A physician purchased the following quantities of medicine, at three different times: 1℔. 4℥. 5ʒ.; 3℔. 11℥. 6ʒ. 2℈. 15gr.; and 7ʒ. 1℈. 12gr.; what was their whole weight? Ans. 5℔. 5℥. 3ʒ. 1℈. 7gr.

(7.) Bought at one time 7bu. 3pk. of wheat; at another, 9bu. 1pk.; and had previously in each of two bins, 6bu. 3pk.; what was the whole amount? Ans. 30bu. 2pk.

(8.) Bought four casks of wine, of which the first contained 42gal. 2qt. 1pt.; the second, 65gal. 1pt.; the third, 50gal. 3qt.; and the fourth, 55gal. 1qt. 1pt.; how many gallons were there in all? Ans. 213gal. 3qt. 1pt.

(9.) Received from France 12 ells, 4qr. of broadcloth; 17 ells, 5qr. 1na. of cassimere; and 19 ells, 2qr. 3na. of silk; how many ells were there in all?
Ans. 50 ells.

(10.) What is the weight of the compound formed from the following ingredients: 5℔. 2℥. 3ʒ. 1℈. 12gr. of calomel; 3℔. 10℥. 5ʒ. 15gr. of jalup; 7℔. 8℥. 7ʒ. 2℈. 14gr.

of rhubarb; and 1℔. 3℥. 2ʒ. 15gr. of the extract of colocynth? Ans. 18℔. 1℥. 2ʒ. 2℈. 16gr.

(11.) A goldsmith bought 4 ingots of silver, the first of which weighed 8lb. 2oz. 12pwt.; the second, 5lb. 4oz. 5pwt.; the third, 6lb. 10oz. 11pwt.; and the fourth, 6lb. 11oz. 15pwt.; what was the weight of the whole?

Ans. 27lb. 5oz. 3pwt.

(12.) A farmer has due from one man £241 12s. 9d. 2qr.; from another, £241 17s. 6d.; from a third, £211 19s. 6d.; what is due from all?

Ans. £695 9s. 9d. 2qr.

(13.) What is the weight of the following silver articles: one spoon weighing 1lb. 10oz. 19pwt.; one tankard weighing 4lb. 6oz.; one basin weighing 1lb. 10oz. 19pwt.?

Ans. 8lb. 3oz. 18pwt.

(14.) Add together the following parcels: 12cwt. 2qr. 17lb.; 1 ton, 19cwt. 3qr. 27lb. 15oz. 15dr.; 2 tons, 12cwt. 3qr. 20lb. 12oz. 10dr.

Ans. 5 tons, 5cwt. 2qr. 9lb. 12oz. 9dr.

(15.) What is the weight of three loads of coal, weighing as follows: the first, 9cwt. 3qr. 12lb.; the second, 10cwt. 1qr. 12lb.; the third, 8cwt. 3qr. 27lb.?

Ans. 29cwt. 0qr. 23lb.

(16.) The distance from A to B is 3 M. 6 fur. 27 rd.; from B to C, 1 L. 2 M. 7 fur.; from C to D., 27 L. 1 M. 2 fur. 39 rd. I demand the distance from A to D.

Ans. 30 L. 2 M. 0 fur. 26 rd.

(17.) A ship sailed south 6 L. 1 M.; S.SE. 9 L. 2 M. 0 fur. 39 rd.; E.SE. 21 L. 1 M. 6 fur. 7 rd. What was the whole distance sailed. Ans. 37 L. 1 M. 7 fur. 6 rd.

(18.) How many French Ells in the following pieces of cloth: the first, 22qr. 3na.; the second, 31qr. 1na.; the third, 50qr. 2na.; the fourth, 91qr. 1na.?

Ans. 32 F. Ells, 3qr. 3na.

(19.) A man had several lots of land; the first contained 27A. 3 sq. r.; the second, 19A. 2 sq. r. 27 sq. rd. 17 sq. ft.; the third, 6A. 2 sq. r. 39 sq. rd. Required the quantity in the three lots. Ans. 54A. 26 sq. rd. 17 sq. ft.

(20.) What is the sum of the following sticks of timber: the first, 3 tons, 49 s. ft. 1672 s. in.; the second, 4 tons, 47 s. ft. 1000 s. in.; the third, 6 tons, 40 s. ft. 1001 s. in.?

Ans. 15 tons, 38 s. ft. 217 s. in.

(21.) Add 3 cords, 4 cord feet of wood, 12 solid feet; 6 cords, 5 cord feet of wood, 12 solid feet; 9 cords, 6 cord feet of wood together.* Ans. 20 cords, 8 s. ft.

(22.) Add together 64A. 2 sq. r. 39 sq. rd.; 61A. 3 sq. r. 17 sq. rd.; 104A. 2 sq. r. 22 sq. rd.
Ans. 231 A. 38 sq. rd.

(23.) What is the sum of the following: 19gal. 2qt. 1pt.; 12gal. 1qt.; 21gal. 3qt.; and 1hhd. 3gal.?
Ans. 1hhd. 56gal. 2qt. 1pt.

(24.) Add together the following: 123 P. 1hhd. 63gal. 3qt. 1pt.; 43 P. 1hhd. 40gal. 2qt.; 4 P. 40gal.
Ans. 172 P. 18gal. 1qt. 1pt.

(25.) Add together the following parcels: 1627bu. 3pk. 7qt. 1pt.; 972bu. 2pk. 1qt.; 2471bu. 1pk. 2qt. 1pt.
Ans. 5071bu. 3pk. 3qt.

(26.) The following quantities of wheat were sold by a merchant: 41ch. 30bu. 3pk.; 51ch. 35bu. 1pk.; 60ch. 7bu. What was the quantity sold? Ans. 154ch. 1bu.

(27.) Add together 12yr. 6mo. 3wk. 6da. 23hr. 12min. 12sec.; 49yr. 5mo. 2wk. 4da. 12hr. 9min.; 1yr. 9mo. 1wk. 1da. 1hr. 6min.; the year being Julian.
Ans. 63yr. 8mo. 3wk. 5da. 12hr. 27min. 12sec.

(28.) Add 21yr. 9mo. 6da.; 31yr. 6mo. 21da.; 6yr. 4mo. 9da.; 12yr. 7mo. 10da. together.
Ans. 72yr. 1mo. 18da.

§ 113. 1. CASE VIII. *To multiply a denominate fraction by a simple number.*

RULE. I. Write the multiplier under the lowest denomination of the multiplicand.

II. Multiply the lowest denomination in the multiplicand; divide the product by the number of that denomination which makes one of the next higher; write down the remainder, and retain the quotient to be added to the product of the next higher denomination.

III. Multiply the next higher denomination in the multiplicand; add to the product the quotient retained from that below it; divide by the number of that denomination which makes one of the next higher; write down the remaider, &c., as before.

* A cord foot consists of 16 solid feet; 8 such feet make a cord.

IV. Proceed in this manner to the last or highest denomination given, which multiply as a simple number.

2. *Illustrations.*

Multiply 4cwt. 3qr. 26lb. 10oz. 12dr. by 8.

cwt.	qr.	lb.	oz.	dr.
4	3	26	10	12
				8
39	3	19	6	0; or 1T. 19cwt. 3qr. 19lb. 6oz.

(1.) Here we first multiply the drams, 12, by 8; the product resulting is 96. As 16 drams make an ounce, or a unit of the next higher denomination, we find there are six such units in this quantity, or 6 ounces, without remainder. We therefore write a cipher in the plaee of drams, and retain the 6oz. to be added to the product of the next higher denomination.

(2.) We then multiply the 10oz. by 8, and add to the product the 6oz. retained from the lower denomination. We have then 86oz., in which there are 5lb. and 6oz. over. We write the 6oz. in their proper place, and retain the 5lb. to be added to the product of the next multiplication.

(3.) We proceed in this manner, regarding the law of increase from right to left in avoirdupois weight, till we come to the highest denomination, which is multiplied as a simple number.

3. *Explanation.* The peculiarity in this multiplication is the same as that in the preceding addition (§ 112. 3), and requires no other explanation. Any abreviations, which are practicable, that are made use of in simple multiplication, may be applied here.

4. *Examples.*

(1.) How much wood in 27 loads, each containing 1 cord 18 s. ft? Ans. 30 c. 102 s. ft.

(2.) What will be the weight of 12 silver cups, each weighing 1 lb. 1 oz. 1 pwt. 20 gr? Ans. 13 lb. 1 oz. 2 pwt.

(3.) What will 700 bushels of potatoes cost, at 1s. 8d. a bushel? Ans. £43 15s.

(4.) If a person travel 32 rd. 3 ft. 10½ in. in a minute, how far will he go, at that rate, in 2 hours?
Ans. 12 m. 28 rd.

(5.) If a span of horses eat 2bu. 3pk. of oats in one week, how many will they eat in 25 weeks? Ans. 68bu. 3pk.

(6.) How much sugar in 12 barrels, each containing 3 cwt. 2qr. 27lb? Ans. 2T. 4cwt. 3qr. 16lb.

(7.) How much water will be contained in 96 hogsheads, each containing 62gal. 1qt. 1pt. 1gi.? Ans. 5991gal.

(8.) What is the cost of 117cwt. of raisins, at £1 2s. 3d. per cwt. £130 3s. 3d.

(9.) If I have 9 fields each of which contains 12A. 2sq. r. 25sq. rd.; how many acres are there in the whole?
Ans. 113A. 3sq. r. 25sq. rd.

(10.) An apothecary had 3 appentices; and he directed that each one should take 35℔. 9℥. 4ʒ. 2℈. 15gr. of medicine, and mix it; how much was there mixed?
Ans. 107℔. 4℥. 6ʒ. 2℈. 5gr.

(11.) A merchant purchased 27 pieces of broadcloth; each piece contained 19yd. 3qr. 1na. What was the quantity purchased? Ans. 534yd. 3qr. 3na.

(12.) Suppose one ship is in 5° 15′ 45″ south latitude, and another 4 times farther south: what must be the latitude of the latter? Ans. 21° 3′ south latitude.

(13.) If a man drink 1 pint 2 gills of ale, for 29 successive days, what quantity will he have drank in all?
Ans. 5gal. 1qt. 1pt. 2gi.

(14.) If a soldier's ration of bread be 5lb. 6oz. 8dr. per week, what will it amount to in 52 weeks?
Ans. 2cwt. 2qr. 1lb. 2oz.

(15.) What cost 94 casks of cider, at 12s. 2d. per cask?
Ans. £57 3s. 8d.

(16.) How many yards in 34 pieces of cloth, each piece containing 27 yards 3qr. Ans. 943yd. 2qr.

(17.) A merchant bought 95 pairs of shoes, at 4s. 6d. 1qr. a pair; how much did he pay for the whole?
Ans. £21 7s. 9d. 3qr.

(18.) A gentleman bought 43 silver spoons, each weighing 2oz. 14pwt. 6gr.; what was the weight of the whole?
Ans. 9lb. 8oz. 12pwt. 18gr.

§ 114. 1. We have regarded Denominate Fractions in two lights; as units, the subdivisions of which have a constant relation to each other, so that a certain number of units in one subdivision form a unit in another, which is higher (§ 102. 1, 5); and as fractions, of which each lower denomination is a part or parts of a preceding or higher denomination, the denominators to which are conventional and always implied (§ 102. 1).

2. These two views have thus far coalesced, and not been distinguished either in the operations or results.

3. We may however distinguish between them; and, regarding denominate expressions in the first view, treat each subdivision as a unit which may be again subdivided according to a conventional system, each subdivision having a fixed relation to that which precedes; so that the numbers may regularly decrease from the highest unit, through all the subdivisions below that unit, precisely analagous to what has been done in decimals.

4. Denominate expressions of lineal measures when required to be multiplied by the same, which produce surfaces, and these again by lineal measures, which produce solids, we treat agreeably to this view. Indeed, multiplication is limited, in its application to denominate expressions, to such as produce things really existing in nature.

5. Hence linear feet and inches may be multiplied together, and a surface is the result; or feet and inches, expressing surface, may be multiplied by linear feet and inches, and a solid is the result.

6. But such quantities as money into money, and weight into weight, or money into weight, &c., being incapable of producing any result known in nature, cannot properly be subject to multiplication as such.

7. The foot then, for multiplication, is subdivided conventionally, so that the subdivisions are in a regular decrease below it as the whole. These subdivisions are in twelves; so that we have a kind of fractions in twelves, corresponding to the *decimals* in tenths. These are called *duodecimals*, and their multiplication is called the multiplication of duodecimals, or cross multiplication.

8. The denominations of lineal and superficial dimensions, as duodecimals, are the foot, the inch or prime, the second, the third, the fourth, the fifth, &c.

Subdivisions of the foot, lower than the inch, are rarely used.

One foot equals 12 inches or primes, which are marked with a single accent ′; one prime equals 12 seconds, marked with a double accent ″; one second equals 12 thirds, and so on; the several denominations, after feet, being distinguished by one, two, &c., accents; as 11ft. 4′ 7″ 6‴ 3⁗.

9. The multiplication of other denominate expressions by the same, as money into weight or measure, &c., when required, is performed not by multiplication properly as such; but by regarding the lower subdivisions in the multiplier as fractional parts of a whole, or determined unit; and taking such fractional parts of the multiplicand as are most easy for division, which are sometimes called *aliquot* parts; and finally adding together the results. This method is usually called *Practice*.

§ 115. 1. CASE IX. *To multiply a denominate fraction expressing one or two dimensions, in feet, and subdivisions of the foot in primes, seconds, &c., by another of the same kind expressing one dimension.*

Or, shorter; To multiply duodecimals.

RULE. I. Write the multiplier under the corresponding denominations of the multiplicand.

II. Multiply, as in the last case, each denomination in the multiplicand, beginning with the lowest, by the highest denomination in the multiplier, and write the result of each under its respective term.

III. Then multiply in the same manner by the next denomination in the multiplier, and write each result one place to the right of the denomination multiplied in the multiplicand.

IV. Multiply in the same manner by the next denomination in the multiplier, and write each result two places to the right of the denomination multiplied in the multiplicand.

V. Proceed in the same manner with all the rest of the denominations in the multiplier.

VI. The sum of the several products will be the required product.

2. *Illustration.* Multiply 3ft. 9′ by 2ft. 7′.

$$\begin{array}{lll} \text{f.} & & \\ 3 & 9' & \\ 2 & 7 & \\ \hline 7 & 6 & \\ 2 & 2 & 3'' \\ \hline 9\text{f.} & 8' & 3'' \end{array}$$

(1.) Here we multiply each denomination in the multiplicand, beginning with the lowest, by 2f., the highest denomination in the multiplier. The 9′, considered with reference to the denomination of feet, are $\frac{9}{12}$f., and their multiplication by 2f.—the foot being the determined unit of the kind of quantity—may be regarded as the multiplication of a fraction by a whole number (§ 76. 1). The result then is $\frac{18}{12}$f., which equals 1f. and 6′. The 6′ are written under the primes, in the multiplicand which produced them; and the 1f. is retained to be added to the product next above.

(2.) We then multiply by the 7′. This considered with reference to feet, is $\frac{7}{12}$f. The multiplication is therefore that of $\frac{9}{12}$f. by $\frac{7}{12}$f. (§ 77. 1), which gives $\frac{63}{144}$; or $\frac{63}{12}$ of $\frac{1}{12}$, or 63″, which equals 5′ 3″, writing down the 3″ one place to the right of the term in the multiplicand from which it was produced, we reserve the 5′, which are $\frac{5}{12}$f. to be added to the product of 3f. by 7′

(3.) The 3f. are next multiplied by the 7′, and the product is that of 3f. by $\frac{7}{12}$f. (76. 1), or $\frac{21}{12}$f.; to which we add the 5′ or $\frac{5}{12}$f. reserved, and we have $\frac{26}{12}$f., or 2f. and 2′. Writing down these, as in the illustration, and taking the sum of the products thus obtained, we have for the product of 3f. 9′ by 2f. 7′, 9f. 8′ 3″.

3. *Explanation.* The exposition of duodecimals given above (§ 114.), and the illustration, sufficiently elucidate all the principles in this operation.

4. The result required in an operation of this kind may often be more concisely and conveniently obtained, by changing at once the lower denominations to the fraction of the highest (§ 108), and multiplying the factors as vulgar

fractions. Thus, 2f. 8′ by 2f. 8′, would be $2\frac{8}{12}$ or $2\frac{2}{3}$f. by $2\frac{2}{3}$f., which would be

$$\tfrac{8}{3}\text{f.}\times\tfrac{8}{3}\text{f.}=\tfrac{64}{9}\text{f.}=7\text{f.}+\tfrac{1}{9}\text{f.}=7\text{f.}+1'+\tfrac{3}{9}'=7\text{f. }1'\,4''.$$

5. The same too may be done by decimals.

6. The instructor will require the pupil to solve the examples which follow, the three ways; and in practice that should be used which in a given case shall seem to be best.

7. *Examples.*

(1.) How many square feet in a board 17 feet 7 inches long, and 1 foot 5 inches wide? Ans. 24ft. 10′ 11″.

(2.) How many cubic feet in a stick of timber 12ft. 10in. long, 1ft. 7in. wide, and 1ft. 9in. thick? Ans. 35ft. 6′ 8″ 6‴.

(3.) How many cubic feet of wood in a load 6ft. 7in. long, 3ft. 5in. high, and 3ft. 8in. broad? Ans. 82ft. 5′ 8″ 4‴.

(4.) Bought a load of wood, which was 3ft. wide, 2ft. 8in. high, and 8ft. long; what part of a cord of wood did it contain? Ans. half a cord.

(5.) A load of wood was 4ft. 6in. wide, 3ft. 10in. high and 7ft. 8in. long; how many feet more than a cord did it contain? Ans. $4\frac{1}{4}$ft.

(6.) There is a house with 4 tiers of windows, and 4 windows in each tier; the height of the first tier is 6ft. 8′; of the second, 5ft. 9′; of the the third 4ft. 6′; of the fourth 3ft. 10′; and the breadth of each is 3ft. 5′; how many square feet do they contain in the whole? Ans. 283ft. 7′.

(7.) What is the number of square feet in a marble slab whose length is 5ft. 7in. and breath 1ft. 10in? Ans. 10ft. 2in. 10″.

(8.) What is the content of a ceiling 43ft. 3′ long, and 25ft. 6in. broad? Ans. 1102ft. 10′ 6″.

(9.) How much wood in a cubic pile measuring 8ft. on every side? Ans. 4 cords.

(10.) What is the price of a marble slab, whose length is 5 feet 7 inches, and breadth 1 foot 10 inches, at one dollar per foot. Ans. \$10,23+.

(11.) There is a house with 3 tiers of windows, 3 in a tier, the height of the first tier is 7ft. 10in.; of the second 6ft. 8in.; and of the third, 5ft. 4in., and the breadth of each 3ft. 11in. What will the glazing come to, at 14d. per foot. Ans. £13 11s. 10½d.

(12.) What is the price of a load of grain, the box containing it being 7ft. 10in. long, 3ft. 6in. broad, and 2ft. high, at $1,25 a bushel? Ans. $54,247+.

(13.) What is the price of a load of coal, the box containing it being 11ft. 8in. long, 3ft. 6in. broad, and 3ft. 6in. high, at $0,10 per dry bushel? Ans. $11,4112+.

§ 116. 1. The remaining case in multiplication of denominate fractions (§ 114. 9) involving division, will be deferred to be placed after division (§ 120. 1). We now give, therefore,

§ 117. 1. Case X. *To subtract denominate fractions.*

Rule. I. Write the subtrahend under the corresponding denominations of the minuend.

II. Begin at the right or lowest denomination, and subtract, in each column successively to the highest, the lower number from the upper; where the upper number is too small for the lower to be taken from it, increase it by one taken from the next higher column, changed to the parts of that lower; then, in subtracting the next column, remember that the upper number has thus been diminished one.

III. Subtract the last or highest column as whole numbers.

2. *Illustration.*

	cwt.	qr.	lb.	oz.	dr.
From	176	0	25	10	13
Take	37	2	11	14	9
Rem.	138	2	13	12	4

(1.) Here we begin at the lowest column and subtract 9dr. from the 13 above it, and we obtain the remainder 4dr.

(2.) We next endeavor to subtract the 14oz. from the 10oz., which cannot be done; we therefore increase the

10oz. by 1 taken from the pounds, which, being equal to 16oz., added to the 10oz., gives 26oz. From this we subtract the 14oz. and obtain the remainder 12oz.

(3.) In the next column, that of pounds, we have then only 24lb., one having been taken from it and added to the next lower column, from which we subtract the 11lb. and obtain the remainder 13lb.

(4.) Then we endeavor to subtract the 2qr. from 0qr., which cannot be done; we therefore increase the 0qr. by 1 taken from the cwt.; from which, it being equal to 4qr., we subtract the 2qr. and we obtain the remainder 2qr.

(5.) We then have only 175cwt., one having been taken from the cwt. and added to the next lower column, from which we subtract the 37 as in whole numbers.

3. *Explanation.* The peculiarity of this subtraction is the same as that in the preceding addition and multiplication, and needs no further explanation.

4. *Examples.*

(1.) A silversmith had 34lb. 9oz. 10pwt. of silver; he melts 19lb. 15pwt. 10gr.; how much has he left?
Ans. 15lb. 8oz. 14pwt. 14gr.

(2.) If out of 6℔. 10℥. 6ʒ. 2℈. of medicine, be taken 4℔. 5℥. 4ʒ. 1℈. 17gr., what quantity will remain?
Ans. 2℔. 5℥. 2ʒ. 0℈. 3gr.

(3.) Bought 145yd. 3qr. of cloth, and sold 95yd. 2qr. 3na.; how much remains? Ans. 50yd. 1na.

(4.) A farmer had 500A. 1 sq. r. of land; one of his sons having married, he gave him 150A. 3 sq. r. 25 sq. rd.; how much had he remaining?
Ans. 349A. 1 sq. r. 15 sq. rd.

(5.) Out of a granary which contained 283bu. of corn, there was taken 153bu. 2pk. 5qt.; what quantity remains?
Ans. 129bu. 1pk. 3qt.

(6.) A merchant bought 8cwt. 2qr. 17lb. of sugar, and sold the same week 5cwt. 3qr. 25lb. How much remains unsold? Ans. 2cwt. 1qr. 20lb.

(7.) Borrowed £50 10s.; paid again at one time, £17 11s. 6d., at another time, £9 4s. 8., at another time, £17 9s. 6d., and at another time, 19s. 6d. 2qr. How much remains unpaid? £5 4s. 9d. 2qr.

(8.) Suppose a footman goes 3m. 4fur. 17rd. an hour, and a rail-road car 39m. 2fur. 20rd. in the same time; how much does one gain on the other in an hour?

Ans. 35m. 6fur. 3rd.

(9.) From 17T. of round timber take 1720 solid inches.

Ans. 16T. 39ft. 8 s. in.

(10.) A grocer bought 5hhd. of molasses, and sold 1hhd. 25gal.; how much had he then on hand?

Ans. 3hhd. 38gal.

(11.) Suppose a father's age is 45yr. 6mo. 3wk. 5da., and his son's 22yr. 9mo. 1wk. 6da.; how much does the father's age exceed the son's?

Ans. 22yr. 9mo. 1wk. 6da.

(12.) The moon is 5S. 18° 14′ 17″ east of the sun, and jupiter 12S. 28° 43′ 45″; how far are the moon and jupiter apart? Ans. 7S. 10° 29′. 28″.

(13.) Suppose 315 cords 68ft. of wood be taken from a pile containing 1000 cords; how many cords will be left?

Ans. 684 cords 60 feet.

(14.) A merchant bought 500yd. 2na. of broadcloth, and sold 412yd. 2qr.; how much had he left?

Ans. 87yd. 2qr. 2na.

(15.) From a pipe of wine a merchant sold to one man, 31gal. 2qt.; to another, 5gal.; and to a third, 3gal. 1qt. 1pt.; how much remains of the pipe?

Ans. 86gal. 1pt.

§ 118. 1. Case. XI. *To divide a denominate fraction by a simple number.*

Rule. I. Write the divisor and the dividend as in division of whole numbers.

II. Begin at the left or highest denomination of the dividend, divide it by the divisor, and write the result in the quotient.

III. If there be a remainder after the first division, change it to the next lower denomination (§ 104.); add to it that lower denomination in the dividend; divide the sum by the divisor; and write the result, which is of that lower denomination, one place to the right of the number in the quotient already found.

IV. Proceed in the same manner through all the de-

nominations to the lowest; and the whole quotient thus obtained will be the result required.

2. *Illustration.*

	£	s.	d.
9)	120	16	6
	13	8	6

(1.) Here we divide the £120 by 9 and obtain the quotient £13, and a remainder £3.

(2.) We then change the £3 to shillings, to which we add the 16s. in the dividend, and divide the sum 76s. by 9, and obtain the quotient 8s., and a remainder 4s.

(3.) We then change the 4s. to pence, to which we add the 6d in the dividend, and divide the sum 54d. by 9, and obtain the quotient 6d.

(4.) Thus we have the entire result, £13 8s. 6d.

3. *Explanation.* In this division there is no principle involved with which the pupil has not already been made familiar.

4. If the number is larger than twelve, long division may be employed; or if the divisor be a composite number, the division by it may be as in simple numbers (§ 42. 1).

5. *Examples.*

(1.) If it takes 2700yd. of broadcloth to clothe a regiment of 800 men, what quantity will each man require?
Ans. 3yd. 1qr. 2na.

(2.) Suppose a poor man labors a month for 1cwt. 1qr. 9lb. 13oz. of pork; how much does he receive each day, on an average, allowing 26 days to each month?
Ans. 5lb. 12oz. $3\frac{1}{13}$dr.

(3.) If a hogshead of wine cost £45 8s. 3d., what is it worth a gallon? Ans. 14s 5d.

(4.) Bought 2 dozen silver spoons, which weighed 7lb. 6oz. 13pwt.; how much silver did each spoon contain?
Ans. 3oz 15pwt. 13gr.

(5.) Suppose 37 barrels of equal size contain 98bu. 3pk. 2qt. of wheat; what quantity is in each barrel?
Ans. 2bu. 2pk. $5\frac{11}{37}$qt.

(6.) Suppose a steamboat, in making 121 trips from Albany to New-York, occupies 48da. 17hr. 40min.; what will be the average time in which she makes one trip?

Ans. 9hr. 40min.

(7.) Eleven men own equal shares of 36hhd. 42gal. 2qt. of wine; what is each man's share?

Ans. 3hhd. 21gal. $1\frac{5}{11}$gi.

(8.) If 259A. 1sqr. r. 10sq. rd. of land be divided into 18 equal lots, how much land will be contained in a lot?

Ans. 14A. 1sq. r. 25sq. rd.

(9.) Bought 17cwt. 3qr. 19lb. of sugar, and sold out $\frac{1}{3}$ of it; how much remains unsold?

Ans. 11cwt. 3qr. 22lb.

(10.) Seven men bought 16hhd. 24gal. 3qt. of wine, for which they paid £45 18s. 6d.; each man paying the same money and consequently entitled to an equal share of wine. What was each man's share, and how much money did he pay? Ans. His share was 2hhd. 21gal. $2\frac{1}{7}$qt.

He paid £6 11s. 2d. $2\frac{4}{7}$qr.

(11.) If 16 men cut 53 cords 69ft. of wood in 2 days, what did each man cut per day. Ans. 1 cord $86\frac{5}{32}$ft.

§ 119. 1. The principles of denominate numbers, limiting their multiplication to those expressing in a certain manner certain kinds of quantity (§ 114.), apply with still greater force to their division.

2. It is totally impossible that denominations of money, or of money and weight, &c., should be divided the one by the other, and a quotient be obtained in the same denominations.

3. All that can be desired by a division of such quantities is to ascertain how many times one denomination or class of denominations, may be contained in others of the same kind. And the result can be obtained not by their direct division, but by their being first changed to the lowest denomination given, and then treated as simple or abstract numbers; and the quotient as such a number would then show the relation of one to the other, and that only.

4. Or, the denominate fractions may be changed to decimals of the highest denomination; and by their division the same result be obtained as by the other method.

5. But in the case of lineal dimensions, which produce by their first multiplication superficial magnitudes, and by a second solids, as both objects really exist in nature, the division of one such denominate fraction by another expressing lineal dimension is possible; and a quotient may be obtained in the given denominations.

6. The process, however, is too complicated for practical use; a better method therefore, to execute such a division, is to change both dividend and divisor to the lowest denomination contained in either, and then divide; as is done in Case V., changing a denominate to a vulgar fraction; or change both to decimals (§ 109. 1) and then divide.

7. Exercises under these remarks and applications of the principles, can be supplied by the teacher.

§ 120. 1. CASE XII. *To multiply a denominate fraction by the same expressing fractional parts of the whole;*
Or, to perform the operations of Practise.

RULE. I. Multiply first by the highest number or unit in the multiplier.

II. Then distribute the lower or fractional parts of the multiplier into such as are most easily taken; take the parts of the multiplicand which are indicated by them; and write the results under the corresponding denominations of the first product.

III. The sum of the parts and of the first product will be the result required.

2. *Illustration.* What will 6cwt. 3qr. 14lb. of sugar cost, at £3 15s. 6d. per cwt. ?

3qr.=$\frac{1}{2}+\frac{1}{4}$cwt.	£.	s.	d.	
14lb.=$\frac{1}{8}$cwt.	3	15	6	
			6	
	22	13	0=6cwt.	
	1	17	9=$\frac{1}{2}$ "	
		18	10	2=$\frac{1}{4}$cwt.
		9	5	1=$\frac{1}{8}$ "
	£25	19	0	3

(1.) Here we first multiply the £3 15s. 6d.—the price per cwt.—by 6, the highest number, or units in the multiplier; as by case VIII.

(2.) We then resolve the 3qr. into the fractions of the denomination above, $\frac{1}{2}$ and $\frac{1}{4}$; and take the parts of the multiplicand denoted by them, and write them under the corresponding denominations of the first product.

(3.) Next we change the 14lb. to a fraction of the highest denomination, $\frac{1}{8}$cwt., and take the part of the multiplicand denoted by it.

(4.) Having now multiplied by each part of the multiplier, (for it will be seen that the taking of parts of the multiplicand is the multiplication of a fraction by a fraction (§ 77. 1)), we add the parts and the first product together, and obtain the result required, £25 19s. 3qr.

3. *Explanation.* The exposition of the character of this multiplication already given (§ 114. 9), renders explanation almost unnecessary.

4. It is proper, however, here to remind the pupil that in the first multiplication, that by the highest number or units of the multiplier, that number is regarded as a simple or abstract number; and that thus a rational product is obtained, and that it could be so in no other way;—the absurdity of saying, 6cwt. times £3 15s. 6d., for instance, is obvious.

5. The multiplication by the lower parts of the multiplier, has already been stated to be simply the multiplication of a fraction by a fraction (§120. (4)). This is apparent from the principle of this multiplication, which regards all the lower denominations in either of the given numbers, as fractional parts of the whole, or highest denomination (§ 114. 9).

6. For the distribution of the fractional parts of the multiplier into those most convenient and most easily taken, no special directions can be given; it must be left to the intelligence and dexterity of the pupil.

7. The great object is, in all cases, to obtain the required result in the most concise manner. The pupil therefore should task his powers to obtain this unaided.

8. He will often find that when a part of the whole is wanted, and he already has a part from which the part

wanted may easily be taken, that it will be expedient to take a part of that part for the required part of the whole (§ 64. 5.).

9. But, we say again, a choice of methods must be left to the pupil; and as he can acquire facility and accuracy in no class of arithmetical operations, without much exercise, so is this especially true here.

10. Another and better method of solving all questions of the character here involved, will be presented in Part IV. Yet it is well for the pupil to be acquainted with these operations, that he may, in a given instance, make his own choice.

11. *Examples.*

(1.) What will be the cost of 55bu. 3pk. 5qt. of wheat at 10s. 2d. 3qr. per bushel? Ans £28 11s. 10d $1\frac{31}{32}$qr.

(2.) What will 5yd. 3qr. 2na. of broadcloth cost at £2 3s. 8d. per yard? Ans. £12 16s. 6d. 2qr.

(3.) What is the value of 5cwt. 3qr. 14lb. of sugar at £2 4s. 6d. per cwt? Ans. £13 1s. 5d. 1qr.

(4.) What is the value 7cwt. 3qr. 12lb. of tobacco at £5 7s. 8d. per cwt? Ans. £42 5s. 11$\frac{3}{[illegible]}$d.

(5.) What is the value of 37T. 14cwt. 2qrs. 14lb. of hemp at £89 6s. 8d. per ton? Ans. £3370 13s. 2d.

(6.) What will 57cwt. 3qr. 8lb. of cordage cost at £3 17s. 9d. per cwt? Ans. £223 16s. 2d.

(7.) What will 27T. 16cwt. 2qr. 18lb. of iron cost at £90 10s. per ton? Ans. £2520 0s. 5d+.

(8.) What will 25A. 2sq. r. 25sq. rd. of land cost at $29 per acre? Ans. $744,03+.

(9.) What will 121yd. 2qr. of broadcloth cost at $0,71 per yard? Ans. $86,265.

(10.) What will 13lb. 10oz. 12pwt. 16gr. of silver cost at £4 7s. 6d. per pound? Ans. £60 14s. 11$\frac{1}{2}$d.

(11.) What will 24lb. of tallow cost at $11,91 per cwt.? Ans. $2,552+.

(12.) What will 1cwt. 3qr. 14lb. of raisins cost at £2 11s. 8d. per cwt? Ans. £4 16s. 10d. 2qr.

(13.) What will 1cwt. 1qr. 8lb. of sugar cost at $8,65 per cwt? Ans. $11,42.

(14.) What will 362$\frac{1}{2}$ bushels of wheat cost at $1,12$\frac{1}{2}$ per bushel? Ans. $407,53.

(15.) What will 27½gal. of brandy cost at $1,25 per gallon? Ans. $34,37½.

(16.) What will 60 bushels of apples cost at 16⅔ cents per bushel? Ans. $10.

(17.) What will 75½ bushels of potatoes cost at $0,33⅓ per bushel? Ans. $25,16⅔.

(18.) What will 46⅜lb. butter cost at 12½ cents per lb.? Ans. $5,79⅔.

(19.) What will 1gal. 2qt. 1pt. of wine cost at $3,62½ per gallon? Ans. $5,89.

(20.) What will 1bu. 3pk. 6qt. of beans cost at $1,12½ per bushel? Ans. $2,18.

(21.) What will 29¾yd. calico cost at $0,20 per yd.? Ans. $5,95.

(22.) What will 27½yd. of silk cost at $1,12½ per yd.? Ans. $30,93¾.

(23.) What will 1cwt. 16lb. of iron cost at $6,75 per cwt.? Ans. $7,71⅞.

(24.) What will 24lb. of sugar cost at $11,25 per cwt.? Ans. $2,41.

(25.) What will 1000 quills cost at ½ct. a piece? Ans. $5.

(26.) The interest on a certain sum for a year being $17,60, what is it for 7mo. 20da.? Ans. $11,24⅘.

§.121. 1. While the United States were colonies of Great Britain, their currency was sterling. Each colony issued its own money in bills, the denominations to which were the pound, as the unit, with its subdivisions of shillings, pence and farthings. Some time after the separation of the colonies from the mother country, their bills depreciated; and in different degrees, in the different colonies. Hence arose an inequality in the value of the pound and its subdivisions in the different states.

2. When therefore, the Federal Currency was adopted in 1786, the pound and its subdivisions in the several states had a different value in that currency; in some more and in others less.

The fraction of a pound equalling a dollar, varied therefore, accordingly.

3. The value of the pound and its subdivisions in all the new states, that is, in the states added to the confeder-

acy since 1786, is the same as that in New-York and North Carolina.

I. In English or Sterling Money;

£1=20s.=240d. $1=4s. 6d.=54d.; therefore,

£1=$$\frac{240}{54}$=$$\frac{40}{9}$.

$1=£$\frac{54}{240}$=£$\frac{9}{40}$.

II. In Canada, Nova Scotia, and New-Brunswick;

£1=20s. $1=5s.; therefore,

£1=$$\frac{20}{5}$=$$\frac{4}{1}$.

$1=£$\frac{5}{20}$=£$\frac{1}{4}$.

III. In New-England, Virginia, Kentucky, and Tennessee;

£1=20s. $1=6s.; therefore,

£1=$$\frac{20}{6}$=$$\frac{10}{3}$.

$1=£$\frac{6}{20}$=£$\frac{3}{10}$.

IV. In New-York, North Carolina, and the new states;

£1=20s. $1=8s.; therefore,

£1=$$\frac{20}{8}$=$$\frac{5}{2}$.

$1=£$\frac{8}{20}$=£$\frac{2}{5}$.

V. In Pennsylvania, New-Jersey, Delaware and Maryland;

£1=20s.=240d. $1=7s. 6d.=90d.; therefore,

£1=$$\frac{240}{90}$=$$\frac{8}{3}$.

$1=£$\frac{90}{240}$=£$\frac{3}{8}$.

VI. In South Carolina and Georgia;

£1=20s.=240d. $1=4s. 8d.=56d.; therefore,

£1=$$\frac{240}{56}$=$$\frac{30}{7}$.

$1=£$\frac{56}{240}$=£$\frac{7}{30}$.

§ 122. 1. To ascertain the comparative value of the unit or pound in the various currencies with regard to some fixed standard of estimation, and to change the denominations of one to those of another, are operations very frequently required.

2. The uniform standard of estimation is the dollar, the unit in Federal money. Its value, in parts of a pound, in the different currencies, is furnished in the tables; so is the value of the pound in parts of the dollar.

3. By the aid of the tables then—which the pupil should in every case be required to form for himself—and of the principles which they recognize, operations on the currencies are rendered comparatively clear and simple.

4. The three cases which follow, embrace all that is necessary for guidance in regard to such operations generally.

§ 123. 1. CASE XIII. *To change pounds, shillings, pence and farthings, in the different currencies, to Federal Money.*

RULE. Change all the lower denominations given to the decimal of a pound (§ 109. 1); divide by that fraction of a pound which equals a dollar in the required currency; and the quotient will be the result.

2. *Illustration.* Change £4 8s. 6d. English currency to Federal money.

12)6, d.	£$\frac{9}{40}$ English money	= \$1
	£4 8s. 6d.	= £4,425
20)8,5 s.	£4,425÷$\frac{9}{40}$	= \$19,666+.
£0,425=8s. 6d.		

(1.) Here we first change the lower denominations, the shillings and the pence, to the decimal of a pound, which we annex to the pounds, and obtain £4,425.

(2.) We then divide this result by $\frac{9}{40}$, the fraction of a pound sterling, which equals a dollar, and we have \$19,666+, the result required.

3. *Explanation.* The principles involved in this operation are simply those of changing a denominate to a decimal fraction (§ 109.); and of division by a fraction;

with all which, at this stage, the pupil is supposed to be familiar.

4. It is not necessary that the lower denominations should be changed to the *decimal* of a pound, as directed in the rule; they may be changed to that denomination and be in the form of a vulgar fraction (§ 108. 1); but the former mode is shorter and more convenient.

5. The divisor may be changed to a decimal, if preferred in that form. It will be well for the pupil to solve the examples in both ways.

6. *Examples.*

(1.) Change £45 15s. 7½d. New-England currency to Federal money. Ans. $152,604+.

(2.) Change £463 10s. 8d. New-England currency to Federal money. Ans. $1545,111+.

(3.) Change £35 19s. Virginia currency to Federal money. Ans. $119,833+.

(4.) Change £345 10s. 11d. 1qr. New-Hampshire currency to Federal money. Ans. $1151,8229+.

(5.) Change £214 10s. 7½d. New-York currency to Federal money. Ans. $536,328+.

(6.) Change £304 11s. 5d. North Carolina currency to Federal money. Ans. $761,427+.

(7.) Change £468 New-York currency to Federal money. Ans. $1170.

(8.) Change £105 14s. 3¾d. New-York currency to Federal money. Ans. $264,289+.

(9.) Change 17s. 9¾d. North Carolina currency to Federal money. Ans. $2,226+.

(10.) Change £417 14s. 6d. Georgia currency to Federal money. Ans. $1790,25.

(11.) Change £94 14s. 8d. South Carolina currency to Federal money. Ans. $405,998+.

(12.) Change £160 Georgia currency to Federal money. Ans. $685,714.

(13.) Change £100 South Carolina currency to Federal money. Ans. $428,571+.

(14.) Change £41 17s. 9d. South Carolina currency to Federal money. Ans. $179,514+.

(15.) Change 11s. 6d. Georgia currency to Federal money. Ans. $2,464+.

(16.) Change £140 10s. Georgia currency to Federal money. Ans. \$602,142+.

(17.) Change £245 New Jersey currency to Federal money. Ans. \$653,33⅓.

(18.) Change £36 11s. 8½d. Pennsylvania currency to Federal money. Ans. \$97,561+.

(19.) Change £25 3s. 7d. Maryland currency to Federal money. Ans. \$67,144+.

(20.) Change £100 Delaware currency to Federal money. Ans. \$266,66+.

(21.) Change £99 7s. 6½d. Delaware currency to Federal money. Ans. \$265,005+.

(22.) Change £125 8s. Maryland currency to Federal money. Ans. \$334,40.

(23.) Change £1 2s. 6d. Canada currency to Federal money. Ans. \$4,50.

(24.) Change £241 18s. 9d. Nova Scotia currency to Federal money. Ans. \$967,75.

(25.) Change £528 17s. 8d. Nova Scotia currency to Federal money. Ans. \$2115,53+.

(26.) Change £224 19s. New-Brunswick currency to Federal money. Ans. \$899,80.

(27.) Change £50 English money to Federal money. Ans. \$222,22+.

(28.) Change £36 10s. 9d. English money to Federal money. Ans. \$162,33⅓.

(29.) Change £1003 10s. English money to Federal money. Ans. \$4460.

(30.) Change £20 18s. 5¾d. New-England currency to Federal money. Ans. \$69,746+.

(31.) Change £240 New-Jersey currency to Federal money. Ans. \$640.

(32.) Change £58 13s. 6½d. Canada currency to Federal money. Ans. \$234,70+.

§ 124. 1. CASE XIV. *To change Federal Money to pounds, shillings, pence and farthings, in the different currencies.*

RULE. Multiply the given number by the fraction of a pound which equals a dollar in the required currency, and the product will be the result.

2. *Illustration.* Change \$84 to Sterling money.

£$\frac{9}{40}$ Sterling money equals a dollar.

\$84$\times\frac{9}{40}$=£$\frac{756}{40}$=£18 18s.

Here we multiply the given number \$84, by $\frac{9}{40}$, the fraction of a pound sterling which equals a dollar, and we obtain £$\frac{756}{40}$, which equals £18 18s. (§ 110. 1), the required result.

3. *Explanation.* The operations in this case are the multiplication of a whole number by a fraction (§ 76. 1); and the changing a vulgar to a denominate fraction (§ 110. 1); the principles of which do not require repetition here.

4. The multiplier may be changed to a decimal, before the multiplication, if preferred.

5. *Examples.*

(1.) Change \$629 to New-York currency.
Ans. £251 12s.

(2.) Change \$152,60 to New England currency.
Ans. £45 15s. 7,2d.

(3.) Change \$65,36 to South Carolina currency.
Ans. £15 5s.

(4.) Change \$110,511 to New Jersey currency.
Ans. £41 8s. 9¾d.

(5.) Change \$36,11 to North Carolina currency.
Ans. £14 8s. 10¼d.

(6.) Change \$196 to Kentucky currency.
Ans. £58 16s.

(7.) Change \$690,45 to Georgia currency.
Ans. £161 2s 1,2d.

(8.) Change \$2907,56 to Nova Scotia currency.
Ans. £726 17s. 9½d.

(9.) Change \$315,44 to Maryland currency.
Ans. £118 5s. 9½d.

(10.) Change \$75 to English money.
Ans. £16 17s. 6d.

(11.) Change \$2114,50 to Canada currency.
Ans. £528 12s. 6d.

(12.) Change \$425,07 to Virginia currency.
Ans. £127 10s. 5d.

(13.) Change $575 to English money.
Ans. £129 7s. 6d.

(14.) Change $75,965 to South Carolina currency.
Ans. £17 14s. 6d.

(15.) Change $152,62 to New England currency.
Ans. £45 15s. 8½d.

(16.) Change $67,144 to Pennsylvania currency.
Ans. £25 3s. 6¾d.

(17.) Change $311,75 to New Brunswick currency.
Ans. £77 18s. 9d.

(18.) Change $918,50 to English money.
Ans. £206 13s. 3d.

(19.) Change $45 to Virginia currency.
Ans. £13 10s.

(20.) Change $741 to Nova Scotia currency.
Ans. £185 5s.

(21.) Change $361,10 to Delaware currency.
Ans. £135 8s. 3d.

(22.) Change $198,67 to New-York currency.
Ans. £79 9s. 4½d.

§ 125. 1. Case XV. *To change the denominations of pounds, shillings, pence and farthings, of one currency to those of another.*

Rule. I. Make the value of a dollar, in denominations of the given currency, the denominator to a fraction of which the value of a dollar, in denominations of the required currency, is the numerator.

II. Multiply the given denominations by the fraction thus obtained, and the product will express their value in the currency required.

2 *Illustrations.* (1.) Change £8 7s. 5d. New-England currency to that of New-York.

	£	s.	d.
$1=8s. N. Y. currency.			
$1=6s. N. E. currency.	8	7	5×1⅓
Therefore, the multiplying fraction is $\frac{8}{6}=\frac{4}{3}=1\frac{1}{3}$.	2	15	9 2⅔
	11	3	2 2⅔

(2.) Change £11 3s. 2d. $2\frac{2}{3}$qr. New-York currency to that of New-England.

$1=6s. N. E.
$1=8s. N. Y.
Therefore, the fraction to multiply by is $\frac{6}{8}=\frac{3}{4}$.

£	s.	d.	qr.
11	3	2	$2\frac{2}{3}\times\frac{3}{4}$
33	9	8	0
8	7	5	

(3.) Change £3 4s. 7d. New-Jersey currency to that of Georgia.

$1=4s. 8d.=56d. Georgia.
$1=7s. 6d.=90d. N. J.
Therefore, the fraction to multiply by is $\frac{56}{90}=\frac{28}{45}$.

£	s.	d.
3	4	$7\times\frac{28}{45}$
90	8	4
2	9	$2\frac{2}{9}$

3. *Explanation.* The principle of this operation is simple. The only matter connected with it requiring explanation is that of the fractional multiplier. This is expressive of the value relatively of any unit that may be taken in the two currencies specified; of the pound, the shilling, or the penny. We use it as expressing the relation between pounds; and by means of this relation are able most concisely to perform all operations under the Case.

4. In the illustrations above, we see first that the pound New-England is equal to $1\frac{1}{3}$ pound New-York; second, that the pound New-York is equal to $\frac{3}{4}$ of a pound New-England; and, third, that the pound New-Jersey is equal to $\frac{28}{45}$ of a pound Georgia; and, consequently, that the pound Georgia is equal to $1\frac{17}{45}$ of a pound New-Jersey.

5. *Examples.*

(1.) Change £84 10s. 8d. New-Hampshire currency to that of New-Jersey. Ans. £105 13s. 4d.

(2.) Change £120 8s. 3d. Connecticut currency to that of New-York. Ans. £160 11s.

(3.) Change £120 10s. Massachusetts currency to that of Georgia. Ans. £93 14s. $5\frac{1}{4}$d.

(4.) Change £410 18s. 11d. Rhode Island currency to that of Nova Scotia. Ans. £342 9s. 1d.

(5.) Change £524 8s. 4d. Virginia currency to sterling money. Ans. £393 6s. 3d.

(6.) Change £214 9s. 2d. New-Jersey currency to that of Virginia. Ans. £171 11s. 4d.

(7.) Change £100 New-Jersey currency to that of New-York. Ans. £106 13s. 4d.

(8.) Change £100 Delaware currency to sterling money. Ans. £60.

(9.) Change £116 10s. New-York currency to that of New-England. Ans. £87 7s. 6d.

(10.) Change £112 7s. 3d. Georgia currency to that of New-England. Ans. £144 9s. $3\frac{3}{4}$d.

(11.) Change £100 Canada currency to that of New-England. Ans. £120.

(12.) Change £116 14s. 9d. sterling money to that of New-England. Ans. £115 13s.

(13.) Change £104 10s. Canada currency to that of New-York. Ans. £167 4s.

(14.) Change £100 Nova Scotia currency to that of New-Jersey. Ans. £150.

(15.) Change £200 New-Jersey currency to that of Georgia. Ans. £124 8s. 10d. $2\frac{2}{3}$qr.

(16.) Change £150 New-Jersey currency to English money. Ans. £90.

(17.) Change £200 New-York currency to that of New-Jersey. Ans. £187 10s.

(18.) Change £150 New-York currency to that of Georgia. Ans. £87 10s.

(19.) Change £125 19s. 7d. New-York currency to English money. Ans. £75 17s. 8d. $3\frac{1}{2}$qr.

(20.) Change £214 16s. 8d. New-York currency to that of Canada. Ans. £134 5s. 5d.

(21.) Change £88 2s. 3d. Georgia currency to that of New-England. Ans. £113 5s. 9d.

(22.) Change £100 South Carolina currency to that of New Jersey. Ans. £160 14s. $3\frac{3}{7}$.

(23.) Change £170 8s. 4d. Georgia currency to that of New-York. Ans. £297 2s. $10\frac{2}{7}$d.

(24.) Change £363 1s. 4d. South Carolina currency to English money: Ans. £350 2s.

(25.) Change £431 5s. 2d. Georgia currency to that of Canada. Ans. £462 1s. 3d.

(26.) Change £525 18s. 6d. English money to that of New England. Ans. £701 4s. 8d.

(27.) Change £603 3s. 9d. English money to that of Delaware. Ans. £1005 6s. 6d.

(28.) Change £352 1s. 2d. English money to that of New York. Ans. £625 17s. $7\frac{5}{9}$d.

(29.) Change £100 English money to that of South Carolina. Ans. £103 14s. $0\frac{8}{9}$d.

(30.) Change £423 18s. 9d. English money to that of Canada. Ans. £471 0s. 10d.

(31.) Change £1456 5s. 10d. Canada money to that of New England. Ans. £1747 11s.

(32.) Change £232 3s. 8d. Canada currency to that of New-York. Ans. £371 9s. $10\frac{2}{5}$d.

(33.) Change £333 9s. 8d. Canada currency to that of New Jersey. Ans. £500 4s. 6d.

(34.) Change £1530 16s. 3d. Canada currency to that of Georgia. Ans. £1428 15s. $1\frac{3}{5}$d.

(35.) Change £9834 10s. 10d. Canada currency to English money. Ans. £8851 1s. 9d.

(36.) Troy, Jan. 5, 1841.

Mr. J. R. Smith,

Bought of James Fisher:

8 pair of Worsted Hose,	at 4s. 6d.	$4,50
6 pair of Thread do.,	" 3 6	2,625
3 yd. of Cassimer,	" 12	4,50
6 yd. of Muslin,	" 1 6	1,125
4 Shawls,	" 22	11,00
2 gross Gilt Buttons,	" 18 6	4,625
25 yd. of Irish Linen,	" 10	31,25
2 pieces of Blue Shalloon,	" 57 6	14,375
$28\frac{1}{2}$ yd. of Calico,	" 2 4	8,3125
		$82,3125

Received Payment.

James Fisher.

§ 126. *Miscellaneous Examples.*

(1.) How many seconds in 8S. 12° 14′ 26″?
Ans. 908066″.

(2.) How many inches from Montpelier to Burlington, it being 38 miles? Ans. 2407680in.

(3.) Sold a piece of cloth containing 8 yards, at £2 14s. a yard; and another piece containing 12 yards, at £1 13s. 4d. a yard; what is the amount of the whole?
Ans. £41 12s.

(4.) A farmer has 3 farms: the first contains 125 A. 3 sq. r.; the second, 200 A. 2 sq. r. 18 sq. rd.; the third, 175 A. 10 sq. rd. He intends to divide these farms equally between his two sons; what will be the share of each son? Ans. 250 A. 2 sq. r. 34 sq. rd.

(5.) There are 7 chests of drawers; in each chest there are 18 drawers; and in each drawer there are 6 divisions; in each division there are £16 6s. 8d.; how much money is there in the whole? Ans. £12348.

(6.) What is the difference between $\frac{2}{9}$ of £1 and $\frac{4}{3}$ of a shilling? Ans. 3s. 1d. $1\frac{1}{3}$qr.

(7.) A person at his death left landed estate to the amount of £2000, and personal property to the amount of £2803 17s. 4d. He directed that his widow should receive one eighth of the whole, and that the residue should be equally divided among his four children. What was the widow's and each child's portion?

Ans. { Widow's portion £600 9s. 8d.
Each child's portion £1050 16s. 11d.

(8.) A man traveled $28\frac{3}{7}$ miles the first day, $33\frac{1}{14}$ miles the second day, and $29\frac{1}{21}$ miles the third day; how far did he travel in all? Ans. 90m. 4fur. 15rd. 3ft. $11\frac{1}{7}$in.

(9.) What part of a barrel is $\frac{1}{3}$ of $5\frac{1}{3}$ of $6\frac{1}{4}$ of a pint?
Ans. $\frac{25}{157}$bar.

(10.) If a steamboat in crossing the Atlantic goes 211m. 4fur. 32rd. in a day; how far will she go in 15 days?
Ans. 3174m.

(11.) If a vessel sail 25° 42′ 40″ in 10 days how far will she sail in one day? Ans. 2° 34′ 16″.

(12.) How many miles are $\frac{9}{10}$ of 7 miles, multiplied by $\frac{11}{15}$ of $87\frac{3}{11}$? Ans. $403\frac{1}{5}$m.

(13.) What part of 3 pence is $\frac{2}{3}$ of 2 pence. Ans. $\frac{4}{9}$.

(14.) Change ,0098 of a ton to a denominate fraction.
Ans. 21lb. 15oz. 3,712dr.

(15.) One man bought $\frac{1}{9}$ of $\frac{4\frac{1}{2}}{4\frac{1}{4}}$cwt. of iron, another $\frac{1}{9}$ of $\frac{5\frac{1}{2}}{5\frac{1}{5}}$cwt.; how much more did one buy than the other.
Ans. $3\frac{1201}{1989}$dr.

(16.) Change $\frac{1}{576}$ of a tun to the fraction of a quart.
Ans. $\frac{7}{4}$qt.

(17.) Boston, Jan. 9, 1840.

Mr. ALLEN BLAKE,

Bought of JOHN WILLIAMS:

2	brass Fire Sets,	at 57s.		$
2	doz. Looking Glasses,	" 25	6d.	
½	doz. Shovels,	" 60		
½	doz. Cotton Cards,	" 36		
4	doz. Butt Hinges,	" 6		
3	pair Steelyards,	" 9		
18	setts Knives and Forks,	" 4	6	
3	⋈ cut saws,	" 42		
				$78,50

Received Payment.

JOHN WILLIAMS.

(18.) Philadelphia, June 4, 1840.

Mr. JOHN PALMER,

Bought of JAMES WATSON:

18 lb.	Coffee,	at 1s.	6d.	
14 lb.	Salmon,	"	6	
12 lb.	Sugar,	"	9	
12 lb.	Black Tea,	" 5		
6 lb.	Souchong do.,	" 4	4	
4 gal.	Molasses,	" 4	6	
3 bar.	Cider,	" 20		
9 lb.	Buckwheat,	"	4	
				$28,00

Received Payment.

JAMES WATSON.

(19.) A brewer disposed of the following quantities of ale: 47bar. 28gal. 3qt.; 123bar. 12gal. 2qt.; 17bar. 19 gal. 3qt.; 45bar. 11gal. 1qt. 1pt. What was the amount disposed of? Ans. 234bar. 0gal. 1qt. 1pt.

(20.) Change 2ft. 6in. to a decimal of a yard.
Ans. ,8333+yd.

(21.) Change $\frac{7}{9}$ of a cwt. to a denominate fraction.
Ans. 3qr. 3lb. 1oz. $12\frac{4}{9}$dr.

(22.) What is the freight of a bale containing 65ft. 9in., at $15 per ton of 40ft., or $0,37½ per foot?
Ans. $24,65+.

(23.) The Chinese wall is said to be 1200 miles in length, averaging 18 feet high, and as many thick; how many solid fathoms does it contain?
Ans. 9504000 fathoms.

(24.) How many trees 4ft. apart every way, may grow in a nursery of 1 acre of ground? Ans. 2722 trees.

(25.) Suppose a steamboat, at $119,75 per share, cost $25472,50, and that $\frac{62}{230}$ of it sold for $5725. Was there a gain or loss by the sale? Ans. $1141,50 loss.

(26.) A pile of wood 84ft. 6in. long, 22ft. 7in. high, and 23ft. 10in. wide, is sold at $3,26 per cord; what is the amount? Ans. $1158,339+.

(27.) Required the cost of a lot of land 62ft. $11\frac{3}{4}$in. long and 27ft. 3½in. wide, at $1,80 per square foot?
Ans. $3093,851+.

(28.) How many yards of carpet, yard wide, will cover a floor 25 feet long and 18 feet wide. Ans. 50 yards.

(29.) There is a yard 58ft. 6in. in length, and 54ft. 9in. in breadth. How many dollars will it cost to pave it, at 5d. New-York currency, per square yard?
Ans. $18,488+.

(30.) Change £63 15s. New-England currency to Federal money. Ans. 212,50.

(31.) What will 7cwt. 19lb. of tallow cost at £3 16s. per cwt.? Ans. £27 4s. 10d. 2qr.

(32.) What is the difference between ,41 of a day, and ,16 of an hour? Ans. 9hr. 40min. 48sec.

(33.) What is the sum of ,17 of a lb. troy, and ,84 of an ounce? Ans. 2oz. 17pwt. 14,4gr.

(34.) Change $\frac{1}{17}$ of a lb. troy to the fraction of an ounce.
Ans. $\frac{12}{17}$ oz.

(35.) Divide 371bu. 1pk. of wheat equally among 270 men; what will each receive? Ans. 1bu. 1pk. 4qt.

(36.) A man has to travel 75 miles; he walks the first day 20m. 3fur.; the second, 18m. 5fur. 20rd.; the third, 23m. 7fur. 30rd.; how much of his journey remains every evening?

Ans. { First evening, 54m. 5fur.
Second evening, 35m. 7fur. 20rd.
Third evening, 11m. 7fur. 30rd.

§ 127. The operations which have thus far been presented and required, must have so familiarized the pupil with the combination and decomposition of numbers; have so acquainted him with the two great principles by which all operations in numbers are effected, diminution and increase; and have induced such a degree of mental discipline and such an invigoration of the powers of analysis as to prepare him for higher combinations and less simple deductions.

If this be not so, a review from this point is earnestly recommended. And, in making it, let the pupil realize distinctly that but two principles are involved in all the operations required of him; that of increase, in Addition and Multiplication; that of diminution, in Subtraction and Division.

We first have these connected with simple numbers; then with simple fractions; and lastly, with denominate numbers, that is, representatives of specific quantity. No other principles are applied in all the variety of Cases, and Rules, and pages, which have thus far been presented, or which are ever devoted to the subject of Arithmetic.

This idea realized as it may and should be, will render the mind clear as to the past, and prepare it for higher reflections soon to be presented.

The principles of increase and diminution then, are emphatically *the* principles—the mode of application alone affords variety.

With these remarks, we pass on to higher reflections on quantity as expressed by numbers; and by analysis, to make an application of the principles, in one mode, sufficient for all practical questions properly connected with this subject.

PART IV.

RATIO.

§ 128. 1. Ratio is the quotient which arises from the division of one number or quantity by another of the same kind; and expresses the relation of one to the other.

2. It is obtained by comparison of the two numbers or quantities; that with which the comparison is made forming the numerator of a fraction, of which that compared forms the denominator; or, in terms of division, that with which the comparison is made, forming the dividend, that compared, the divisor. Thus, the ratio of 5 to 15—often noted 5 : 15—is obtained by comparing 5 with 15, by making 15 the numerator or dividend, and 5 the denominator or divisor; $\frac{15}{5}=3$, the ratio.

3. The two numbers used in a comparison, when named together, are called *terms;* when spoken of separately, the former of the two, that compared, is called the *antecedent,* the other the *consequent.*

4. Simple or abstract numbers, or numbers expressing quantity simply without reference to any other, are compared without difficulty; it being usually obvious which is properly antecedent, and which consequent. Thus, the answer to the question, what is the relation of 6 to 8? is evidently $\frac{8}{6}=1\frac{1}{3}$; 6 being the number compared, 8 that with which it is compared. Or, that to the question, what is the relation of a piece of cloth, in which there are 18 yards, to another, in which there are 36 yards? The relation or ratio of the former to the latter being obviously $\frac{36}{18}=2$. Or, that to the question, what is the ratio of \$8 to \$56? is evidently $\frac{56}{8}=7$.

5. But the comparison of other numbers or quantities requires more reflection. Its use, in such cases, being not to

obtain the ratio simply as a ratio, but by it as a means to attain a further result—a final answer to a proposed question—a clear and distinct analysis of the question is necessary, that its true order may be determined.

6. In questions of this kind, two numbers or quantities which are of the same kind or character, are taken for the comparison (§ 128. 1). Their order of arrangement is determined by that number in the question, which is like the final answer desired. The comparison is made with reference to that number.

7. If the answer should be greater than the number in the question of the same denomination with it, it is plain that the larger of the two numbers compared would be the dividend, to furnish the larger quotient; if the desired answer should be less than the number referred to, the smaller of the two numbers compared would be the dividend, to furnish the smaller quotient (§ 48. 4.).

8. Comparison in all cases expresses the ratio between the terms compared; and united to the number embraced in the question, the character of which forms the subject of the demand, and which is like the desired answer, indicates the answer in the form of a compound fraction.

Thus, if a piece of cloth containing 13 yards was sold for $130, what would be the price of a piece of the same cloth containing 18 yards?

The two numbers which are of the same kind in this question, therefore the two for comparison (§ 128. 1), are 13 and 18; the other number in the question, that like the answer sought, is $130. Now, to determine how the comparison should be made between the 13 and 18, whether the smaller or the greater should be the dividend, we regard the condition of the question and the nature of the demand. These show, in this case, that the final answer is to be cost in dollars; and that that answer is to be greater than the cost, or dollars embraced in the question. For, it is plain that 18 yards must cost more than 13. 18 therefore is the number with which the comparison is made, and 13 that compared. The comparison is therefore $\frac{18}{13}$, the larger dividend giving the larger ratio. $\frac{18}{13}$ united with the cost in the question, gives the compound fraction $\frac{18}{13}$ of $130, indicating the final answer required; that is, that 18 yards will cost $\frac{18}{13}$ of $130. Or, $1\frac{5}{13}$, the ratio of the two quantities, is the ratio of their respective cost, and

multiplied by the cost given in the question, must give the required cost of the 18 yards. Thus,

$$\begin{array}{r|l} \cancel{18} & 18 \\ & \cancel{180} \\ & 10 \\ \hline & \$180 \text{ Ans.} \end{array}$$

§ 129. Many of the questions already presented are proper for this mode of solution; and it is proposed to substitute it for that of most of those remaining properly belonging to Arithmetic. It is here introduced in the place usually assigned to Proportions, which, with their principles, are left to the sciences to which they belong.

This is the mode of application of the two great principles of Arithmetic which we have before referred to (§ 127). Its advantages will be apparent on trial.

§ 130. 1. CASE. *To solve questions by ratio.*

RULE. I. Draw a perpendicular line; and write the number embraced in the question which is like the answer sought at its right.

II. If the desired answer be greater than this number, write the greater of the two remaining numbers under it, and the other at its left, on the left of the line.

III. Perform the operations thus indicated in the shortest mode practicable; by canceling, when it can be done; and, when it cannot, by such abbreviations as may be applicable.

2. *Illustrations.*

(1.) If 15 men can do a piece of work in 18 days, how long will it take 27 men to do the same?

$$\begin{array}{r|l} & \text{da.} \\ & \cancel{18}\ 2 \\ \cancel{27} & \cancel{15}\ 5 \\ \cancel{3} & \\ \hline & 10 \text{ days.} \end{array}$$

In this illustration, we see at once from the question that the answer is to be days; therefore, that 18, the number

in the question expressive of that kind of quantity, is to be first written. This is done, and, over it, the sign or mark of days is placed, to prevent mistake as to the denomination of the answer.

15 and 27, terms which are alike, each expressing the same kind of quantity—men—remain for the comparison. An analysis of the whole question is then made; from which it is apparent, that 27 men will do the work in less time than 15. Therefore, the final answer must be less than 18 days, the time in which the 15 men do the work. We require then the smaller ratio. This is obtained by writing the smaller of the two numbers for the dividend, and making of the other the divisor. The whole, as now written, indicates the answer to be $\frac{15}{27}$ of 18.

We then cancel as far as possible, after which there remain on the right of the line 5 and 2, which, multiplied together, give the true result, 10 days.

(2.) How many yards of matting, 2ft. 6in. wide, will cover a floor that is 27ft. long and 20ft. wide?

$$\begin{array}{rr|l} & & \text{yd. l.} \\ & & \cancel{27}\ \ 9 \\ & & \cancel{20}\ \ 4 \\ \text{2ft. 6in.} = 2\tfrac{1}{2}\text{ft.} = \tfrac{5}{2}\text{ft.} & \cancel{5} & 2 \\ \text{1yd.} = \text{3ft.} & \cancel{3} & \\ \hline & & 72 \text{ yards.} \end{array}$$

This illustration is a question in duodecimals properly, but is more conveniently solved in this mode, as is the great mass of questions of that class; so also are those of Practice.

Yards in length is the matter to be obtained; consequently yd. l. is written at the top of the line, on the right, to designate the character of the answer.

But two nnmbers are embraced in the question, and of them the true order of comparison is apparent.

The two dimensions of length and breadth, multiplied together, make up the surface which is to be covered (§ 114. 5); and this is obviously the dividend. The multiplication is indicated by writing one of those dimensions under the other.

2ft. 6in. is the divisor: this is changed to the fraction of

a foot; 2ft. 6in.$=2\frac{1}{2}$ft.$=\frac{5}{2}$ft. We then divide by it, as in division of fractions (§ 82. 1), by multiplying by the denominator, and, for this purpose, writing it at the right of the line, and dividing by the numerator, by writing it at the left.

By canceling, we dispose of the divisor entirely, and change the numbers in the dividend, to be multiplied together, to 2, 4, and 9. Performing the multiplications, we obtain, as the final result, 72 yards.

The result is obtained in yards by dividing by 3, the number of feet in a yard. This 3 is written under the 5 in the divisor, at the left of the line.

(3.) If $\frac{5}{8}$ of a gallon of wine cost £$\frac{5}{8}$, what will $\frac{5}{9}$ of a tun cost?

$$\begin{array}{r|l}
 & £ \\
\cancel{8} & \cancel{5} \\
\cancel{9} & 5 \\
3 & \cancel{1}\cancel{2}\cancel{6} \\
 & 2 \\
\cancel{5} & \cancel{8}\ \ 42 \\
\hline
3 & 420=£140
\end{array}$$

In this illustration, it is seen at once that the answer is to be cost in money. This is therefore designated by the sign, as before; and £$\frac{5}{8}$ is first written. As this is to be multiplied, the numerator is written at the right, and the denominator at the left of the line (§ 76. 1); and would be read, if necessary, five eighths.

By analysis, we see that $\frac{5}{9}$ of a tun of wine must cost more than $\frac{5}{8}$ of a gallon. $\frac{5}{9}$T. is therefore the dividend. It is written like £$\frac{5}{8}$, with the numerator at the right of the line, and the denominator at its left. As the number to be compared with it is in a lower denomination, gallons, it requires to be changed to the same, that the numbers compared may be of the same kind (§ 128. 1.). This is done by multiplication of it by 2, the number of pipes in a tun, and by 126, the number of gallons in a pipe (§ 104. 1. 6. 8).

The number compared, therefore the divisor, is $\frac{5}{8}$ gal. Its denominator is written at the right, to multiply by, and the numerator at the left, for a divisor (§ 82. 1).

Canceling as far as possible, and performing, on the numbers which remain, the operations indicated, we have as the final result 420 divided by 3, equaling £140.

Attention to this and the preceding illustrations will show how vulgar fractions are in all cases to be disposed, when they occur in either or all the numbers embraced in the question.

A reference to multiplication and division of fractions will sufficiently bring to mind all the principles involved in such operations.

(4.) If 3 yards of cloth may be bought for \$12,75, how many yards may be bought for \$102?

$$\begin{array}{r|l}
 & \text{yd.} \\
 & 3 \\
\cancel{12{,}75} & \cancel{102{,}00} \\
\cancel{425} & \cancel{2040} \\
\cancel{85} & 408 \\
17 & \\
\hline
17 & 408 = 24 \text{ yards.}
\end{array}$$

This illustration need not be detailed; the answer requires to be yards, and greater than the number of that denomination in the question. Hence the analysis, indicating the final result, gives $\frac{10200}{1275}$ of 3.

The decimal places are made equal on both sides of the line, that the numbers for the comparison may be of the same kind (§ 128. 1), and in accordance with the rule for division of decimals (§ 99. 1. Rule. III.).

In all cases, the decimal places on both sides of the line should be made equal; then they may be treated as simple or abstract numbers. The ratio of the dividend and divisor is not thus changed (§ 52. 1).

3. *Explanation.* Distinct explanation for the principles involved in these operations seems unnecessary here. Each step having been made clear in the illustrations, and the reason for this mode of solution having been stated, it can hardly be necessary again to repeat the principles involved. The pupil must be so familiar with them all as not to require it.

4. *Examples.*

(1.) At the rate of 9 yards for £5 12s., how many yards of cloth can be bought for £44 16s.? Ans. 72yd.

(2.) If 240 bushels of wheat are purchased at the rate of \$$22\frac{1}{2}$ for 18 bushels, and sold at the rate of \$$33\frac{3}{4}$ for $22\frac{1}{2}$ bushels, what is the profit on the whole? Ans. \$60.

(3.) At \$2,25 per Ell E., how many yards of cloth can be bought for \$136,80? Ans. 76yd.

(4.) If 12 bushels of wheat be bought for \$13,32, how many can be bought for \$51,06? Ans. 46 bushels.

(5.) If it require 90 yards of carpeting $\frac{3}{4}$ yard wide, to carpet a floor, how many yards $1\frac{1}{8}$ yards wide would be sufficient? Ans. 60yd.

(6.) If a courier, traveling $13\frac{5}{8}$ hours a day, perform a journey in $35\frac{1}{2}$ days, how long will it require if he travel but $11\frac{9}{10}$ hours a day? Ans. 40da. 15hr. 30min.+

(7.) If 20 men can perform a piece of work in 15 days, how many men must be added to the number, that it may be accomplished in $\frac{4}{5}$ of the time? Ans. 5 men.

(8.) If by working $6\frac{3}{5}$ hours a day, a man can accomplish a job in $12\frac{1}{2}$ days, how many days will be required if he work $8\frac{1}{3}$ hours a day? Ans. $9\frac{9}{10}$ days.

(9.) If A can mow an acre of grass in 6 hours, and B in 8 hours, how much will they jointly mow in 10 hours? Ans. $2\frac{11}{12}$ acres.

(10.) In exchange for 120 bushels of wheat valued at \$$1\frac{1}{2}$ per bushel, A receives of B \$65 in cash, and the balance in oats at \$0,40 a bushel; what quantity of oats does he receive? Ans. $287\frac{1}{2}$ bushels.

(11.) If A can mow $\frac{1}{5}$, B $\frac{1}{6}$, and C $\frac{1}{8}$ of an acre of grass in an hour, in how many hours can they together mow $3\frac{1}{5}$ acres? Ans. 6hr. 30min. $30\frac{1}{2}$sec.

(12.) How many yards 3qr. wide are equal to 30 yards 5qr. wide? Ans. 50 yards.

(13.) If \$49,75 gain \$$3,48\frac{1}{4}$ in a year, what principal will gain \$7 in the same time? Ans. \$100.

(14.) What will 26 yards of cloth cost, if \$6,90 are paid for 13 Ells French? Ans. \$9,20.

(15.) If 795 yards of cloth cost \$107,50, how many yards can be bought, at the same rate, for \$427,50? Ans. $3161\frac{22}{43}$ yards.

(16.) The annual wages of a man being \$100, to be paid in land at \$6 per acre, how many acres will he receive after 3 years and 7 months? Ans. $59\frac{13}{18}$ acres.

(17.) If a staff $4\frac{1}{2}$ feet in length, cast a shadow 6 feet, what is the height of a tree whose shadow measures 108ft.

Ans. 81 feet.

(18.) How many men must be employed to finish in 9 days, what 15 men would do in 30 days?

Ans. 50 men.

(19.) The clothing of a regiment of foot soldiers of 750 men amounts to £2831 5s., what will it cost to clothe a body of 3500 men? Ans. £13212 10s.

(20.) If $\frac{3}{4}$ of a yard of velvet cost 7s. 3d., how many yards can be bought for £13 15s. 6d.?

Ans. 28yd. 2qr.

(21.) If A and B together can do a piece of work in 7 days, and B alone in 12 days, in how many days can A alone do $\frac{2}{3}$ of it? Ans. $10\frac{4}{5}$ days.

(22.) If the penny loaf weigh 9oz. when the price of a bushel of wheat is 6s. 3d., what ought it to weigh when wheat is 8s. $2\frac{1}{2}$d. per bushel? Ans. 6oz. 13dr.

(23.) A wall that is to be built to the height of 27 feet, was raised 9 feet in 6 days by 12 men; how many men must be employed to finish the wall in 4 days at the same rate of working? Ans. 36 men.

(24.) What quantity of shalloon that is $\frac{3}{4}$ of a yard wide, will be sufficient to line $7\frac{1}{2}$ yards of cloth $1\frac{1}{2}$ yards wide?

Ans. 15 yards.

(25.) I owned $\frac{2}{5}$ of a ship, and sold $\frac{5}{8}$ of my share for \$780. What was the value of the whole?

Ans. \$3120.

(26.) What is the value of $\frac{3}{4}$ of an ounce of silver, if 2 ounces be valued at $12\frac{2}{3}$ shillings? Ans. 4s. 9d.

(27.) If 7cwt. 1qr. of sugar cost \$64,96, what will be the price of 4cwt. 2qr.? Ans. \$40,32.

(28.) Suppose 120 seamen are provided with 7200 gallons of water for a cruise of 4 months; what is each man's share per day? Ans. 2 quarts.

(29.) If 60 gallons of water in one hour fall into a cistern containing 300 gallons, and by a pipe in the cistern 35 gallons run out in an hour; in what time will it be filled?

Ans. 12 hours.

(30.) A owns $\frac{1}{3}$ of a farm of 108 acres, B $\frac{1}{6}$ of it, C $\frac{1}{9}$, and D the remainder. If D sold his part for \$714, what is the farm valued at? Ans. \$1836.

(31.) A farmer sold 17 bushels of barley and 13 bushels of wheat, for \$31,55; the wheat at \$0,35 a bushel more than the barley; what was the price of each per bushel?

Ans. { Barley, \$0,90.
Wheat, \$1,25.

(32.) If 57lb. 7oz. of spices be bought for \$17,25, what must be paid for 87lb. 10oz. 7pwt.? Ans. \$26,32+.

(33.) If $58\frac{1}{2}$ gallons of wine cost \$28,50, what will 620 gallons cost? Ans. \$302,05+.

(34.) If 562 yards of linen cost \$495 dollars, how many yards can be bought for \$1051?

Ans. 1193yd. 1qr. 0+n.

(35.) If upon 52 acres of land 962 bushels of wheat have been harvested, how many bushels would 225 acres yield at the same rate? Ans. 4162bu. 2pk.

(36.) What will be the amount to pay for 652lb. 6oz. of coffee; if for every 57lb. 6oz. I must pay \$15,15?

Ans. \$171,91+.

(37.) If 40 poles in length and 4 in breadth, make an acre, what must be the length to make an acre, when the breadth is 15 poles? Ans. 10 poles 3yd. 2ft.

(38.) A merchant bought 795 yards of cloth for \$107,50; he has still \$457,50 which he wishes to lay out in the same cloth, at the former price; how many yards may he yet purchase? Ans. 3383yd. 1qr. $1\frac{41}{43}$.

(39.) If the matting for the floor of a room 24ft. by 18 cost \$95,60, what will the same matting come to for a room 22 feet in length by 38 in breadth? Ans. \$185+.

(40.) The forage required by a body of cavalry, for a month of 31 days, is 2821cwt. of hay; how much will be needed for the same body for 87 days? Ans. 7917cwt.

(41.) How many pounds of tea can a man buy for \$672, if he buy 651lbs. for \$327,50? Ans. 1335+lb.

(42.) If 21 men could perform a work in 17 days, and 16 men be added to them after the second day, how much time will be saved by it? Ans. $6\frac{18}{37}$da.

(43.) The annual wages of a man being \$100, to be paid in land at \$6 per acre, how many acres will he receive, after 3 years and 7 months? Ans. $59\frac{13}{18}$ acres.

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(44.) Two men, starting at the same time, ride a certain distance: A travels at the rate of $6\frac{1}{3}$ and B $7\frac{3}{4}$ miles an hour. B completes the journey after 20 hours and 20 minutes; in how long will A accomplish the journey?

Ans. 24hr. 52min. $53\frac{13}{19}$sec.

(45.) A and B departed from the same place and travel the same road, but A goes 7 days before B, at the rate of 24 miles a day; B follows at the rate of 30 miles a day; what distance will they travel before B overtakes A?

Ans. 840 miles.

(46.) A courier who travels at the rate of 40 miles a day, had been despatched 6 days, when a second was sent to overtake him in 20 days; how many miles must he travel every day to accomplish it? Ans. 52 miles.

(47.) A and B, 350 miles asunder, started at the same time and traveled the same road to meet each other; A went 4 miles an hour, and B 3; how many miles did each travel, and how many hours were they traveling, before they met?

Ans. A went 200; B 150 m.; and each 50 hr.

(48.) A and B set out together on a journey of 300 miles; A travels 30 miles every 12 hours, and B 32 in 10 hours; how many hours will A travel more than B to perform his journey? Ans. 26 hours 15 minutes.

(49.) A gentleman, in the afternoon, being asked the time of day, replied it was between 2 and 3, and the hour and minute hands were exactly together; what was the time of day? Ans. 2h. 10m. $54\frac{6}{11}$s.

(50.) The hour and minute hands of a watch are exactly together at 12 o'clock; at what time are they together between 6 and 7? Ans. 6h. 32m. $43\frac{7}{11}$s.

(51.) The earth, being 360 degrees in circumference, rolls round on its own axis once in 24 hours; in what time does it move one degree? Ans. 4 minutes.

(52.) How many yards of paper $1\frac{1}{4}$ yard wide, will be sufficient to hang a room 20 yards square, and 4 yards high? Ans. 256 yards.

(53.) If 375 cwt. may be carried 660 miles for a given sum, how many cwt. may be carried 60 miles for the same money? Ans. 4125 cwt.

(54.) An insurance company, consisting of 82 persons, sustains a loss, of which each man's share was \$12; what

would their shares have been, had the company consisted of only 32 persons? Ans. \$30,75.

(55.) If $5\frac{1}{2}$ yards of muslin that is $1\frac{1}{2}$ yard wide will make a dress, how many yards of lining will be required that is but 3 qr. wide? Ans. 11 yards.

(56.) A lot of ground was walled in by 16 men in 6 days; the same being demolished, is required to be re-built in 4 days; how many men must be employed?

Ans. 24 men.

(57.) A person by traveling 12 hours per day, performs a journey of 800 miles in 32 days; how many days will he require to perform the same journey, if he travel 15 hours per day? Ans. $25\frac{3}{5}$ days.

(58.) If 50 gallons of water fall into a cistern of sufficient capacity to contain 230 gallons, in one hour, and by a pipe 35 gallons be drawn off in the same time, how long will it take to fill the cistern? Ans. 15hr. 20min.

(59.) A vessel at sea discharges a cannon, the report of which reaches me in 1 minute 30 seconds. How far distant is she, allowing sound to travel 1142 feet in a second?

Ans. 19 M. 3 fur. $29\frac{1}{11}$ rd.

(60.) Bought 32 yards of muslin, at 6s. 8d. New-York currency, per yard. What was the cost in Federal money? Ans. \$26,666+.

(61.) After observing a flash of lightning, it was 12 seconds before the thunder was heard; how far distant was the cloud from which it came? Ans. 2M. 4fur. $30\frac{6}{11}$rd.

(62.) Perceiving a man at a distance cutting down a tree with an axe, I remarked that 6 of my pulsations passed between seeing him strike and hearing the sound of the blow; what was the distance between us, allowing 70 pulsations to a minute? Ans. $5873\frac{1}{7}$ feet.

(63.) Two travelers, A and B, leave two places, 100 miles distant from each other, at the same time; A travels $6\frac{1}{3}$ miles per hour, and B $7\frac{3}{4}$ miles per hour; what part of the distance will each of them make before meeting?

Ans. A $44\frac{164}{169}$ miles; B $55\frac{5}{169}$ miles;

And what time will they travel before they meet?

Ans. 7hr. 6min. 2+sec.

(64.) How many yards of cloth were there in a piece which cost \$68,60; the price of the yard being to the number of yards as 5 to 7? Ans. 98 yards.

(65.) Of two travelers upon the same road, A travels 5 miles an hour, B 3 miles an hour; when B passes a certain place on the way, A is 13 miles behind him; at what distance will he overtake B? Ans. $32\frac{1}{2}$ miles.

(66.) Two men bought a lottery ticket in partnership; A gave \$9 towards it, B gave \$7; the ticket draws a prize of \$2000; how much will each of them get?

Ans. A \$1125; B \$875.

(67.) The father of a child is 52 years older than the child; his mother 36 years older; and the age of the father is to that of the mother as 4 to 3; what is the age of the child? Ans. 12 years.

(68.) Three workmen can severally do a piece of work in the following times: A in 3 weeks; B in 8 weeks may perform it 3 times; C 5 times in 12 weeks; in what time will they perform the work jointly?

Ans. $\frac{8}{9}$ of a week.

(69.) If A and B together can perform a work in 8 days, and A and C in 9 days, and B and C in 10 days, how many days will it take each to perform the work alone?

Ans. A $14\frac{34}{49}$da.; B $17\frac{23}{41}$da.; C $23\frac{7}{121}$da.

§ 131. 1. Questions very often occur in which several conditions are connected with the two leading numbers for comparison, and so involve more than one comparison. In such cases, the number which is of the same kind with the desired answer is first written, and, after it, the two leading numbers are compared as though no others were embraced in the question. Then the conditions are taken up, two of the same kind at a time, one from the first member of the question, the other from the last, and compared in the same manner, irrespective of all the rest.

When the comparisons are completed, and all the numbers are written, the operation is as simple as in the preceding single comparisons.

When there are several conditions which together make up but one quantity, they may be all considered as one, and thus separate comparisons be rendered unnecessary. Thus, 10 days 6 hours a day, may be treated as 10×6; and so dimensions; as 20ft. long, 8ft. wide, and 4ft. high, may be treated as one magnitude; $20\times8\times4$, making a cube.

2. *Illustrations.*

(1.) If 6 men build a wall 20 feet long, 6 feet high, and 4 feet thick, in 9 days, working 10 hours a day, in what time will 24 men build one 200 feet long, 8 feet high, and 6 feet thick, working 9 hours a day?

This question, complicated in appearance, is yet simple in solution.

It embraces 11 numbers, 8 of which are conditions connected with that with reference to which comparison is made and with the two leading numbers for comparison, which influence the final result, and therefore must be regarded in the operation.

If these conditions were exactly equal in the two members of the question, they would cancel, and the question would consist only in finding in how many days 24 men would perform the work which 6 men do in 9 days.

But the conditions, not being equal, require consideration. They are therefore taken in pairs, one from each member of the question, and compared, each pair independently, and irrespective of all the other numbers in the question, except that which determines the answer.

The question now presented requires then, for its solution, 5 comparisons: 1st, of the men; 2d, of the hours; 3d, of the length; 4th, of the height; 5th, of the thickness: though the last three might be treated as but one thing, the work done—the several dimensions, by multiplication, giving the solid contents of the wall; therefore the walls themselves could be at once compared, without separating the several dimensions which compose them. The order of the comparisons is:

1st. The men; a greater number of men will employ less time; therefore we have $24 : 6 = \frac{6}{24}$.

2d. The hours; a less number of hours requires more days; therefore we have $9 : 10 = \frac{10}{9}$.

3d. The length; greater length requires more time; therefore we have $20 : 200 = \frac{200}{20}$.

4th. The height; greater height requires more time; therefore we have $6 : 8 = \frac{8}{6}$.

5th. The thickness; greater thickness requires more time; therefore we have $4 : 6 = \frac{6}{4}$.

Or, taking the last three together, we have,

$$20\times6\times4 : 200\times8\times6 = \frac{200\times8\times6}{20\times6\times4}.$$

These, united with the time in the question, 9 days, with reference to which they are compared, afford successively;

1st. $\frac{6}{24}$ of $9=\frac{6\times9}{24}$.

2d. $\frac{10}{9}$ of $\frac{6\times9}{24}=\frac{10\times6\times9}{9\times24}$.

3d. $\frac{200}{20}$ of $\frac{10\times6\times9}{9\times24}=\frac{200\times10\times6\times9}{20\times9\times24}$.

4th. $\frac{8}{6}$ of $\frac{200\times10\times6\times9}{20\times9\times24}=\frac{8\times200\times10\times6\times9}{6\times20\times9\times24}$.

5th. $\frac{6}{4}$ of $\frac{8\times200\times10\times6\times9}{6\times20\times9\times24}=\frac{6\times8\times200\times10\times6\times9}{4\times6\times20\times9\times24}$.

The number of days resulting from the concurrence of all these circumstances is obtained by performing the operations thus indicated. This is at once done thus:

$$\begin{array}{r|l}
 & \text{da.} \\
\cancel{3} & \cancel{9} \\
\cancel{24} & \cancel{6} \\
\cancel{9} & \cancel{10}\ 5 \\
\cancel{20} & \cancel{200} \\
\cancel{6} & \cancel{8}\ 10 \\
\cancel{2}\ \cancel{4} & \cancel{6}\ \cancel{2} \\
\hline
 & 50 \text{ days.}
\end{array}$$

Hence it is concluded, that the number of days sought in this question, and, generally, the result desired in questions of this character, is equal to the product of all the ratios which result from a comparison of the terms relating to each circumstance or condition of the question, by the number in the question which is like the answer sought, and with reference to which each comparison is made.

(2.) If a man travel 273 miles in 13 days, traveling only 7 hours a day, how many miles will he travel in 12 days, if he travel 10 hours a day?

	M.
	$\cancel{273}$
$\cancel{13}$	12
$\cancel{7}$	10
	$\cancel{21}$
	3
	360 miles.

(3.) If a cellar 22,5ft. long, 17,3ft. wide, and 10,25ft. deep, be dug in 2,5 days, by 6 men, working 12,3 hours a day; in how many days should 9 men dig another 45ft. long, 34,6ft. wide, and 12,3ft. deep, working 8,2 hours a day?

	da.
	$\cancel{2,5}$
$\cancel{3}$ $\cancel{9}$	6
$\cancel{2}$ $\cancel{8,2}$	$\cancel{12,3}$ $\cancel{41}$
$\cancel{22,5}$	$\cancel{45,0}$ 2
$\cancel{17,3}$	$\cancel{34,6}$ $\cancel{2}$
$\cancel{10,25}$	$\cancel{12,3}$
$\cancel{41}$	$\cancel{41}$
	12 days.

The decimals are here first made equal, by annexing ,0 to 45, at the right of the line; then all are canceled except the 2 and 6 at the right; these multiplied together give 12, the answer.

The remarks above, and the illustrations, will guide the pupil to the solution of any questions involving double or compound ratios, which may arise.

3. *Examples.*

(1.) If 8 men make 24 rods of wall in 6 days, how many men will build 18 rods in 3 days? Ans. 12 men.

(2.) If a pasture of 16 acres will feed 6 horses for 4 months, how many acres will feed 12 horses for 9 months? Ans. 72 acres.

(3.) If the wages of 6 men for 14 days be $84, what will be the wages of 9 men for 11 days? Ans. $99.

(4.) If 154 bushels of oats serve 14 horses for 44 days, how long would 406 bushels last 7 horses? Ans. 232 days.

(5.) If 25 men can earn $6250 in 2 years, how long will it take 5 men to earn $11250?

Ans. 18 years.

(6.) If a barrel of beer last 7 persons 12 days, how much will be drank by 42 persons in a year?

Ans. 182½ barrels.

(7.) If 32 men build a wall 36 feet long, 8 feet high, and 4 feet wide in 4 days, in what time will 48 men build a wall 864 feet long, 6 feet high, and 3 feet wide?

Ans. 36 days.

(8.) If by traveling 6 hours a day, at the rate of 4½ miles an hour, a man perform a journey of 540 miles in 20 days, in how many days, traveling 9 hours a day, at the rate of 4⅔ miles an hour, will he travel 600 miles?

Ans. 14²⁄₇ days.

(9.) If a footman in 12 days, traveling 6 hours a day, perform a journey of 240 miles, in how many days will he perform one of 720 miles, if he travel 8 hours a day?

Ans. 27 days.

(10.) If 20 men in 12 days, working 5 hours a day, can perform a piece of work, how many hours a day must 15 men work, in order to perform 3⅓ times as much work in 30 days? Ans. 8⁸⁄₉ hours.

(11.) If 8 men spend $48 in 24 weeks, how much will 40 men spend in 48 weeks at the same rate?

Ans. $480.

(12.) If a eistern 8¾ feet long, 5¼ feet wide, and 6½ feet deep, contain 68¼ barrels of water, how many barrels would it hold, if each of its dimensions were doubled?

Ans. 546 barrels.

(13.) If 1000lb. of wool would make 752 yards of cloth 1⅞ yards wide, what quantity of cloth 2 yards wide, may be made of 1672lb. of the same wool?

Ans. 1178¹⁹⁄₂₅ yards.

(14.) If 248 men, in 5 days of 11 hours each, dig a trench 230 yards long, 3 wide, and 2 deep, in how many days of 9 hours in length, will 24 men dig a trench of 420 yards long, 5 wide, and 3 deep? Ans. 288⁵⁹⁄₂₀₇ days.

(15.) If ½ cwt. be carried 600⅚ miles for $12½, how far may ¾ cwt. be carried for $30⅚? Ans. 988¹⁄₂₇ miles.

(16.) If $350 in 9 months gain $15, what principal will gain $6 in 12 months? Ans. $105.

(17.) If 10 persons drink $15\frac{3}{5}$ gallons of wine in one week, how much will 16 persons drink in 48 weeks?

Ans. $1073\frac{7}{25}$ gallons.

(18.) If 16 men cut 112 cords of wood in 7 days, how many cords will 24 men cut in 19 days?

Ans. 456 cords.

(19.) If 172 boards, 17 feet 6 inches long, and 14 inches broad, are needed to floor a place, how many would it take 12 feet 6 inches long, and 10 inches broad?

Ans. 337,12 boards.

(20.) If the freight of 5 tierces of salt, each weighing $5\frac{1}{2}$ cwt., 80 miles, cost $28, what will be the freight of 75 sacks of salt, each weighing $2\frac{1}{4}$ cwt., 150 miles?

Ans. $322,15+.

(21.) If 12 men, working 9 hours a day, for $15\frac{5}{9}$ days, are able to execute $\frac{2}{3}$ of a job, how many men may be withdrawn, and the residue be finished in 15 days more, if the laborers are employed only 7 hours a day?

Ans. 4 men.

(22.) If a block of marble 2 feet 6 inches long, 1 foot 9 inches broad, and 1 foot 3 inches thick, weigh 9 cwt. 2 qr., what would it weigh if each of its dimensions were doubled?

Ans. 3 T. 16 cwt.

(23.) If 180 men, working 6 days, each day 10 hours, can dig a trench of 200 yards long, by 3 yards wide, and 2 yards deep; how many days will it take 100 men to dig a trench of 360 yards long, 4 wide, and 3 deep, working 8 hours a day?

Ans. 48,6 days.

(24.) A hare is 50 leaps before a greyhound, and he takes 4 leaps while the greyhound takes 3; but 2 greyhound's leaps are equal to 3 hare's leaps; how many leaps must the greyhound make to overtake the hare?

Ans. 300 leaps.

(25.) If 9 men, working 6 days, at the rate of 8 hours per day, can build a wall of 152 feet long, and 9,5 feet high, how many days must 16 men work, at the rate of 10 hours each day to build a wall 295 feet long, and 17,5 feet high?

Ans. $9\frac{3771}{5776}$ days.

(26.) If 352 men, having worked 8 hours every day, have made a certain length of canal in 87 working days, and there remains now $\frac{1}{7}$ of the same length to be done to complete the work, which it is intended they should do in 8

working days; how many hours more per day must they work to complete the task at the same rate of working?

Ans. $6\frac{1}{2}$ hours more.

(27.) If 6 men pave 55 yards of a street in 5 days, how many men will it take to pave 212 yards in 12 days?

Ans. $9\frac{42}{66}$ men.

(28.) A man performing a journey in 21 days by walking 7 hours at the rate of 5 miles an hour, how many days will it take him to perform the same journey by walking 10 hours a day at the rate of $3\frac{1}{2}$ miles an hour?

Ans. 21 days.

(29.) If 5 men, in 10 days, mow 42 acres of meadow, how much will 13 men mow in 18 days?

Ans. $192\frac{14}{25}$ acres.

(30.) If 23 boards, of $12\frac{1}{2}$ feet long and 14 inches broad, make a certain flooring, how many boards will it take of 15 feet long and 10 inches broad? Ans. $26\frac{5}{6}$ boards.

(31.) If 2100 bushels of oats feed 200 horses during 21 days, at $\frac{1}{2}$ a bushel per day, how long will 3700 bushels last 740 horses, at $\frac{3}{8}$ of a bushel per day?

Ans. $12\frac{1}{2}$ days.

(32.) How many yards of paper, 22 inches broad, will cover a wall of 26 yards circuit, and 9 feet high, if 20 yards circuit of the same height can be covered by 72 yards of 30 inch wide paper? Ans. $127\frac{7}{11}$ yards.

(33.) What provision must be made for an army of 9560 men, in bread, if they should receive 2lb. per day for 70 days; if found by experience that 5000 men need, in 25 days, 312500lb. at the rations of $2\frac{1}{2}$lb. per day?

Ans. 2091250lb.

(34.) The common step of a horse being about 4 feet, and that of a man $2\frac{2}{3}$ feet, the man making 8 steps to the horse's 5, how much space will the man gain over the horse, in walking a distance of 18 miles? Ans. $7\frac{1}{2}$M.

§ 132. 1. Questions of per centage are of frequent occurrence in almost every kind of business.

2. The term *per cent.* signifies the hundredth part of either a single unit, or hundred of the kind of quantity considered. Thus, 5 *per cent.* of \$100, means ,05 of \$100, which is \$5; 20 *per cent.* of \$1,00, signifies ,20 of \$1,00, which is \$0,20; 7 *per cent.*, $\frac{7}{100}$ of \$100, or \$7 of a hundred; and 6 *per cent.*, $\frac{6}{100}$ of \$100, or \$6 of a hundred.

3. It is the principle especially on which is computed Commission, Brokerage, Price or Premium for Insurance, Interest and Discount, &c., &c.

4. The solutions of questions of this kind is simple, and governed by the principles just considered.

5. The kind or denomination of the answer wanted, whatever it may be, kept strictly in view, will always suggest the true comparisons.

§ 133. 1. COMMISSION OR BROKERAGE is per centage allowed to an agent for buying or selling goods, or stocks.

2. *Illustration.*

What is the commission on \$4200 at $2\frac{1}{2}$ per cent.?

This question analyzed, is presented, at length, in the form of the questions lately considered; thus, If \$100 give a commission of $2\frac{1}{2}$ per cent., what will \$4200 give? Its mode of solution then is obvious.

The comparison is $\$\frac{42}{100}$ o. $\frac{5}{2}$ per cent.

	com.
$\cancel{2}$	5
$1\cancel{00}$	$\cancel{42}\cancel{00}$
	21
—	—
	105 dollars.

3. *Examples.*

(1.) What is the commission to be allowed for selling goods to the value of \$975, at 8 per cent.? Ans. \$78.

(2.) What is the commission on \$3568 at $1\frac{3}{4}$ per cent.? Ans. \$62,44.

(3.) What is the commission on £843 10s. at $1\frac{1}{4}$ per cent.? Ans. £10 10s. $10\frac{1}{2}$d.

(4.) What is the commission on \$964,90 at $2\frac{1}{4}$ per cent.? Ans. \$21,71.

(5.) What is the commission on \$5425 at 2 per cent.? Ans. \$108,50.

(6.) What is the commission on \$12450,75 at 5 per cent.? Ans. \$622,537.

(7.) What is the commission on \$10000 at $\frac{3}{4}$ of 1 per cent.? Ans. \$75.

(8.) What is the commission on \$827,64 at $2\frac{1}{2}$ per cent.? Ans. \$20,691.

(9.) At $3\frac{1}{2}$ per cent., what must I allow my broker for purchasing \$2525 worth of goods? Ans. \$88,37½.

(10.) At 3 per cent. commission, how much must I allow for selling \$525 worth of goods? Ans. \$15,75.

§ 134. 1. PREMIUM for Insurance is per centage allowed for indemnification against loss.

2. *Illustration.*

What is the premium for the insurance of property valued at \$3664, against loss by fire, at $\frac{3}{4}$ per cent.?

The comparison is $\$\frac{3664}{100}$ of $\frac{3}{4}$ per cent.

	prem.
$\cancel{4}$	3
100	$\cancel{3664}$
	916
100	27,48 dollars.

3. *Examples.*

(1.) If a stock of goods be insured for \$4125 at $\frac{3}{4}$ of 1 per cent., what is the premium? Ans. \$30,937.

(2.) If an insurance of \$25000 be taken on a ship and cargo, returning from Canton to New-York, what is the premium at $4\frac{1}{2}$ per cent.? Ans. \$1125.

(3.) What is the premium for insuring a dwelling-house valued at \$2875, against loss or damage by fire, at $\frac{3}{8}$ of 1 per cent.? Ans. \$10,78.

(4.) What will be the premium for insuring an East India ship and cargo, valued at \$84713,716 at $15\frac{3}{4}$ per cent.? Ans. \$13342,41.

(5.) What is the premium for insuring property valued at \$845, at $\frac{1}{5}$ per cent.? Ans. \$1,69.

(6.) What is the premium for insuring property valued at \$3600, at $\frac{3}{4}$ per cent.? Ans. \$27.

(7.) What would be the premium for insuring a ship and cargo valued at \$18000, at $\frac{3}{8}$ per cent.? Ans. \$67,50.

§ 135. 1. INTEREST is per centage allowed for the use of money. The rate or ratio is determined by the interest on \$100, for one year.

\$100 and the rate per year are therefore embraced in every computation.

2. The rate of interest is fixed by law in the several states. In New-York it is limited to 7 per cent.; and banks are restricted to 6 per cent. on money for 60 days or less; but for more than 60 days they are allowed 7 per cent.

3. Debts due the banks are allowed to stand three days beyond the time at which they are properly due, which are called three days of grace. For these days interest is taken; therefore bank interest is always for three days more than the time specified.

4. *Illustration.*

What is the interest on \$50, for 2 years, at 7 per cent. per annum?

The comparison is \$$\frac{50}{100}$ of $\frac{2}{1}$yr. of \$7.

	interest.
	7
$1\cancel{0}\cancel{0}$	$5\cancel{0}$
	$\cancel{2}$
—	—
	7 dollars.

5. *Examples.*

(1.) What is the interest on \$300 for one year, at 6¼ per cent.? Ans. \$18,75.

(2.) What is the interest on £855 17s. 6d. for one year, at 5¾ per cent.? Ans. £49 4s. 3d.

(3.) What is the interest on £76 for one year, at 5 per cent.? Ans. £3 16s.

(4.) What is the interest on \$326, for one year, at 7 per cent.? Ans. \$22,82.

(5.) What is the interest on £800 for 1 year, at 7 per cent.? Ans. £56.

(6.) What is the interest on \$1256 for 4 years, at 6 per cent. per annum? \$301,44.

(7.) What is the interest on \$1711,51 for 2 years at 5¾ per cent. per annum? \$196,382.

(8.) What is the interest on \$438,25 for 5 years, at 6 per cent. per annum? Ans. \$131,475.

(9.) What is the interest on \$10,15 for 12 years, at 3 per cent.? Ans. \$3,654.

(10.) What is the interest on \$789 for 2 years, at 6 per cent.? \$94,68.

(11.) What is the interest on $37,50 for 4 years, at 6 per cent.? Ans. $9.

§ 136. 1. If the interest be required for years and months, or months alone, they are changed to the fraction of a year, in months.

2. *Illustration.*

What is the interest on $350, for 2 years and 6 months, at 6 per cent.?

2yr. 6mo.=30mo.=$\frac{30}{12}$yr.

The comparison is $$\frac{350}{100}$ of $\frac{30}{12}$yr. of $6.

	interest.
	$\cancel{6}$
$1\cancel{0}\cancel{0}$	$35\cancel{0}$
$\cancel{1}\cancel{2}$	$3\cancel{0}$
2	
2	105
	52,50 dollars.

3. *Examples.*

(1.) What is the interest on £57 17s. 8d. for 3 months, at 6 per cent. per annum? Ans. 17s. 4¼d.

(2.) What is the interest on $453,45 for 3 years and 4 months, at 6 per cent.? Ans. $90,69.

(3.) What is the interest on $7500 for 4 months, at 7 per cent.? Ans. $175.

(4.) What is the interest on £25 for 6 months, at 4 per cent.? Ans. 10s.

(5.) What is the interest on $575 for 8 months, at 6 per cent.? Ans. $23.

(6.) What is the interest on $45 for 6 months, at 8 per cent.? Ans. $1,80.

(7.) What is the interest on $165,45 for 1 year and 6 months, at 5 per cent.? Ans. $12,408.

(8.) What is the interest on $325,41 for 3 years and 4 months, at 5 per cent.? Ans. $54,235.

(9.) What is the interest on $215,34 for 4 years and 6 months, at 3½ per cent.? Ans. $33,916.

(10.) What is the interest on $279,87 for 2 years 6mo., at 7 per cent.? Ans. $48,977.

(11.) What is the interest on $33,50 for 2 years and 6 months, at 5 per cent.? Ans. $4,187.

§ 137. 1. If the interest be required for years, months and days, or for days alone, they are changed to the fraction of a year, in days.*

2. *Illustration.*

What is the interest on $50, for 63 days, at 6 per cent?

63da. $=\frac{63}{360}$yr.

The comparison is $\$\frac{50}{100}$ of $\frac{63}{360}$yr. of $6.

	interest.
	$\cancel{6}$
$\cancel{100}$	$\cancel{50}$ 5
$\cancel{360}$	$\cancel{63}$
40 $\cancel{120}$	21
—	—
	,525 dollars.

3. *Examples.*

(1.) What is the interest on $64,58 for 3 years 5 months and 10 days, at 7 per cent.? Ans. $15,57.

(2.) What is the interest on $789 for 2 years 3 months and 24 days, at 7 per cent.? Ans. $127,95.

(3.) What is the interest on $37,50 for 4 years 11mo. and 18 days, at 7 per cent.? Ans. $13,037.

(4.) What is the interest on $112 for 9 days at, 6 per cent.? Ans. $0,168.

(5.) What is the interest on $234 for 5 days, at 6 per cent.? Ans. $0,195.

(6.) What is the interest on $158 for 2 months and 8 days, at 7 per cent.? Ans. $2,09.

(7.) What is the interest on $575 for 3 years 6 months and 20 days, at 5½ per cent.? Ans. $112,50.

(8.) What is the interest on $213,23 for 3 years and 12 days, at 10 per cent.? Ans. $64,679.

(9.) What is the interest on $49,25 for 3 years 3 months and 3 days, at 6 per cent.? Ans. $9,628.

(10.) What is the interest on $80 for 1 year 5 months and 12 days, at 6 per cent.? $6,96.

(11.) What is the interest on $64,58 for 3 years 5mo. and 10 days, at 6 per cent.? $13,346.

(10.) What is the interest on $984 for 65 days, at 7 per cent.? Ans. $12,436.

(11.) What is the interest on $15650 for 35 days, at 6 per cent.? Ans. $91,29.

*In computing interest, a month is reckoned 30 days; and 12 months a year.

(12.) What is the interest on \$384,50 for 93 days, at 7 per cent.? Ans. \$6,953.

(13.) What is the interest on \$540 for 60 days, at 6 per cent.? Ans. \$5,40.

§ 138. 1. AMOUNT is principal and interest added together.

2. *Illustration.*

What is the amount of \$50, at 6 per cent. for 63 days?

The comparison is \$$\frac{50}{100}$ of $\frac{63}{360}$yr. of \$6.

	amt.
	$\cancel{6}$
20 $\cancel{100}$	$\cancel{50}$
2 $\cancel{6}$ $\cancel{360}$	$\cancel{63}$ 21
40	21,
	,525
	50,
	50,525 dollars.

3. *Examples.*

(1.) What is the amount of \$630 for 8 months, at 6 per cent.? Ans. \$655,20.

(2.) What is the amount of \$764,50 for 3 years and 10 months, at 6 per cent.? Ans. \$940,335.

(3.) What is the amount of \$7342 for 1 year and 4 months, at 6 per cent.? \$7929,36.

(4.) What is the amount of \$450 for 2 years, at 7 per cent.? Ans. \$513.

§ 139. 1. The same principle readily gives the per cent. at which a specified sum will afford a certain amount in a certain time.

2. *Illustration.*

At what per cent. will \$50 give \$57 in two years?

\$57—\$50=\$7 gain.

The comparison is \$$\frac{50}{100}$ of $\frac{1}{2}$yr. of \$7.

	per cent.
	7
$\cancel{50}$	$\cancel{100}$
$\cancel{2}$	1
	7 per eent.

2. *Examples.*

(1.) At what per cent. will \$500 amount to \$650 in 5 years? Ans. 6 per cent.

(2.) At what per cent. will £500 amount to £725 in 9 years? Ans. 5 per cent.

(3.) At what per cent. will \$600 amount to \$856,50 in 9 years and 6 months? Ans. $4\frac{1}{2}$ per cent.

(4.) At what per cent. will \$340,25 amount to \$626,06 in 12 years? Ans. 7 per cent.

(5.) At what per cent. will \$324,61 amount to \$430,108 in 5 years and 5 months? Ans. 6 per cent.

(6.) At what per cent. will \$350 amount to \$360,50 in 9 months? Ans. 4 per cent.

§ 140. 1. The same principle gives the time in which a specified sum will afford a certain amount at a certain rate.

2. *Illustration.*

In what time will \$50 give \$14 interest at 7 per cent?

The comparison is \$$\frac{100}{50}$ of \$$\frac{14}{7}$ of 1yr.

$$\begin{array}{r|l} & \text{time.} \\ & 1 \\ \cancel{50} & \cancel{100}\ 2 \\ \cancel{7} & \cancel{14}\ 2 \\ \hline & 4 \text{ years.} \end{array}$$

3. *Examples.*

(1.) In what time will \$730 amount to \$975,99 at 6 per cent.? Ans. 5 years, 7 mo., 12 days.

(2.) In what time will \$500 double at 7 per cent.? Ans. $14\frac{2}{7}$ years.

(3.) In what time will \$450 double at 6 per cent.? Ans. $16\frac{2}{3}$ years.

(4.) In what time will £540 amount to £734 8s. at 4 per cent.? Ans. 9 years.

(5.) In what time will \$837 amount to \$1029,51 at $5\frac{3}{4}$ per cent.? Ans. 4 years.

(6.) In what time will \$1500 amount to \$2332,50 at 6 per cent.? Ans. 9 years, 3 mo.

(7.) In what time will \$1200 amount to \$1350 at 7 per cent.? Ans. 1yr. 9mo. 13da.

§ 141. 1. It is customary, when payments, in part, are made on a note, bond, &c., to write the sum paid on the *back* of the instrument, which is called *endorsement;* or to give a receipt specifying that it is to be applied in payment of such instrument. The rule for computing interest in such cases, adopted by most of the states, and by the Supreme Court of the United States, is laid down in a decision of Chancellor Kent, of the state of New-York.

According to this decision, the method of casting interest is, to "apply the payment in the first place to the discharge of the interest then due. If the payment exceeds the interest, the surplus goes towards discharging the principal, and the subsequent interest is to be computed on the balance of principal remaining due. If the payment be less than the interest, the surplus of interest must not be taken to augment the principal; but interest continues on the former principal, until the period when the payments taken together exceed the interest due, and then the surplus is to be applied towards discharging the principal; and interest is to be computed on the balance, as aforesaid."

2. *Illustration.*

A note was given February 1, 1830, for the payment of $500, on which there were endorsements as follows: May 1, 1830, $40; November 14, 1830, $8; April 1, 1831, $12; May 1, 1832, $60.

What is the balance due on the note September 16, 1832; interest at 7 per cent?

Principal - - -	$500
Interest to May 1, 1830, (3 m.) - - - -	8,75
First Amount - -	508,75
First payment - - - - - - - - - - -	40,
Balance forming a new principal - - -	468,75
Interest to May 1, 1832, [2 years] - - -	65,62
$8 Second amount -	534,37
$12 =$80 a sum exceeding the interest	
$60 due May 1, 1832. - - -	80,
Balance forming a new principal - - -	454,37
Interest to September 16, 1832, [4½ m.] -	11,92
Balance due on taking up the note - - - -	$466,29

The second and third endorsements not being, singly or together, equal to the interest due at the time they were paid, it is computed from the first payment to the fourth, which, in conjunction with the two previous payments, exceeds the interest then due.

3. *Examples.*

(1.) A has a note dated April 15, 1833, for \$2150,25, on which are the following endorsements: Nov. 8, 1834, \$500,00; September 1, 1835, \$723,64; January 1, 1837, \$378,295; and Oct. 29, 1837, \$850,00. What amount was due on this note, at 6 per cent. interest, April 15, 1838?
Ans. \$138,337.

(2.) What was due on a note of \$2100, dated June 15, 1820, on settlement, June 15, 1830, the following sums being endorsed on the back of it; viz. June 30, 1824, \$750; and September 30, \$1200; interest at 6 per cent.?
\$1249,527.

(3.) A note of hand dated April 4, 1832, was given for the payment of six hundred dollars, on which there were endorsements as follows: July 10, 1832, \$84,60; November 22, 1832, \$10; April 30, 1833, \$14; December 5, 1833, \$309. What was the balance due on taking up the note, April 5, 1834; int. at 7 per cent.? Ans. \$240,95.

(4.) A's note of \$635,84 was dated Sept. 5th, 1837, on which were endorsed the following payments: Nov. 13th, 1839, \$416,08; May 10th, 1840, \$152. What was due March 1st, 1841; the interest being 6 per cent.?
Ans. \$168,01.

(5.) D's note to E for \$1000, with interest at 6 per cent., was dated May 5, 1832, on which the following payments were made: Nov. 17, 1832, \$300; April 23, 1833, \$50; and August 11, 1833, \$520. What was due June 5th, 1834? Ans. \$201,713.

§ 142. 1. When the interest, as it becomes due, is added to the principal, and this amount, as a principal, is left at interest, the interest thence accruing is termed *compound*, because it is interest on interest.

Its definition suggests its mode of computation.

2. *Illustration.*

What will $500 amount to, at compound interest, in five years, at 7 per cent.?

	500,00	principal for 1st year.
$\frac{500}{100}$ of 7 =	35,00	interest do.
	535,00	principal for 2d year.
$\frac{535}{100}$ of 7 =	37,45	interest do.
	572,45	principal for 3d year.
$\frac{572,45}{100}$ of 7 =	40,07	interest do.
	612,52	principal for 4th year.
$\frac{612,52}{100}$ of 7 =	42,88	interest do.
	655,40	principal for 5th year.
$\frac{655,40}{100}$ of 7 =	45,88	interest do.
	$701,28	Amount.
	500,00	
	$201,28	Interest.

Or, when the interest is for years only, as in the present question, thus, for three years:

	amount.
	$\cancel{5}\cancel{0}\cancel{0}$
$1\cancel{0}\cancel{0}$	107
$1\cancel{0}\cancel{0}$	107
$1\cancel{0}\cancel{0}$	107
20	
2000	1225,043
	612,5215 dollars,=amount for three years.

3. *Examples.*

(1.) What is the compound interest of $750, for 4 years, at 6 per cent.? Ans. $223,90.

(2.) What is the compound interest of $500, for 4 years, at 6 per cent.? Ans. $131,23.

(3.) What will $1000 amount to in 4 years, at 7 per cent. compound interest? Ans. $1310,79.

(.4) What is the amount of $425, for 4 years, at 5 per cent. compound interest? Ans. $516,59.

(5.) What is the amount of £400, for 4 years, at 6 per cent. compound interest? Ans. £504 19s. 9¾d.

(6.) What is the compound interest of $450, for 3 years, at 7 per cent.? Ans. $101,269.

(7.) What is the amount of $550,75, for 3 years, at 6 per cent. compound interest? Ans. $655,952.

(8.) What is the compound interest of $729, for 7 years, at 6 per cent.? Ans. $316,78.

(9.) What is the compound interest of $1000, for 5 years, at 6 per cent.? Ans. $338,22.

(10.) What is the amount of $640, for 3 years, at 6 per cent. compound interest? Ans. $762,25.

(11.) What is the compound interest of $295,37, for 2 years, at 6 per cent.? Ans. $36,50.

(12.) What is the compound interest of $100, for 3 years, at 6 per cent.? Ans. $19,101.

§ 143. 1. Discount is an allowance for the payment of money before it is due; it is so much abated as the balance of that money, if put at interest, would give in the same time at the same rate.*

This balance of money is generally termed the *present worth* of the demand.

2. Two steps are usually necessary in Discount: 1st, to find the amount of $1 for the given time, at the given rate; 2d, by means of this, as a divisor, to obtain the present worth.

* In common cases, the interest is taken for the discount, the parties not attending to the real difference between discount and interest. Thus, if $100 be discounted in this way for a year at 6 per cent. $6 is taken out, and the person receives $94. If he were to lend the $94 on interest for a year at the same rate, he would receive of interest $5,64, or 36 cents less than the above discount, which is, in fact, discounting at $6\frac{18}{47}$ per cent., or nearly 6,3 per cent., instead of 6 per cent. The true discount is $5,66, and the person should receive $94,34.

In bank discount, however, the interest is usually considered as the discount.

3. *Illustration.*

What principal will give, as the amount of principal and interest added, $570, in 2 years, at 7 per cent.?

	per cent.		prin.
	7	$ ¢	100
~~100~~	~~2~~	1+14=~~114~~	~~570~~
50			5
	7		500 dollars.
	,14		

4. *Examples.*

(1.) What is the present worth of $161,10 for 19mo., discount at 5 per cent.? Ans. $149,282.

(2.) What is the present worth of $240, one half payable at 4 months, and the other half at 8 months, discount at 5 per cent.? Ans. $234,162.

(3.) What is the present worth of $100, one quarter due in 3 months, and the remainder in 5 months, discount at 7 per cent.? Ans. $97,444.

(4.) Bought goods to the value of $109,64, to be paid in 9 months; what present money will discharge the same, if I am allowed 6 per cent. discount?

Ans. $104,918.

(5.) What is the present worth of $430,67 for 19mo., discount at 5 per cent.? Ans. $399,078.

(6.) What is the present worth of $150, payable in 3 months, discount at 5 per cent.? Ans. $148,148.

(7.) What is the discount on $560, due 9 months hence, at 8 per cent.? Ans. $31,698.

(8.) What is the discount on $50, due 2 years hence, at 12 per cent.? Ans. $9,678.

(9.) What is the present worth of $700, payable in 2 years and 9 months, discount at 7 per cent.?

Ans. $587,002.

(10.) What is the present worth of $500, due 2 years and 8 months hence, discount at 7 per cent.?

Ans. $421,35.

(11.) How much ready money will satisfy a bond of $1000, payable in 6 years and 4 months, discount at 6 per cent.? Ans. $724,637.

(12.) What is the discount of $250, due 3 years and 4 months hence, at 6 per cent.? Ans. $41,666.

(13.) What is the discount of $1000, due 6 years hence, at 6 per cent.? Ans. $246,705.

§ 144. 1. Loss or Gain is per centage above or below cost.

2. *Illustrations.*

(1.) What is the gain per cent. on a yard of cloth which cost $1,02, and was sold for $1,18?

1,18—1,02=,16.

The comparison is $\$\frac{,16}{1,02}$ of $1,00.

$$
\begin{array}{r|l}
 & \text{per cent.} \\
 & \cancel{,16}\ 8 \\
\cancel{1,02} & 1,00 \\
51 & \overline{} \\
 & 800(,15\tfrac{35}{51}\text{ per cent.} \\
 & 51 \\
 & \overline{290} \\
 & 255 \\
 & \overline{35}
\end{array}
$$

(2.) At what must a yard of broadcloth be sold, which cost $4,50, to gain 25 per cent.?

The comparison is $\$\frac{4,50}{1,00}$ of $1,25.

$$
\begin{array}{r|l}
 & \text{amount.} \\
 & 25 \\
\cancel{1,00} & \cancel{4,50} \\
2 & 9 \\
\hline
 & 1,125 \text{ gain.} \\
 & \overline{5,625} \text{ dollars.}
\end{array}
$$

(3.) If a yard of cloth be sold at $5,625, and at that price afford a profit of 25 per cent., what did it cost?

The comparison is $\$\frac{5,625}{1,25}$ of $1,00.

$$
\begin{array}{r|l}
 & \text{per cent.} \\
 & \cancel{1,00} \\
\cancel{1,25} & 5,625 \\
5\ \cancel{25} & \cancel{20}\ 4 \\
\hline
5 & 22,500 \\
 & \overline{4,50} \text{ dollars.}
\end{array}
$$

3. *Examples.*

(1.) If a merchant buy broadcloth at $5,50 a yard, and sell it at $6,60 a yard; what is his profit per cent.?
Ans. 20 per cent.

(2.) A man bought 500 sheep at $2,25 a head, and his expenses in the purchase were $75. He sold them again at an average price of $3,40 a head; what was the profit, per cent. on his investment? Ans. 41$\frac{3}{4}$ per cent.

(3.) A grocer bought tea at 6s. a pound, but in consequence of a fall in the price of the article, is obliged to sell it at 5s. 4d. per pound; what is his loss per cwt.?
Ans. 11$\frac{1}{9}$ per cwt.

(4.) What do I gain per cwt. if I buy wheat at 12s. a bushel, and sell the same for 15s. a bushel?
Ans. 25 per cent.

(5.) Purchased pepper for 8d. a pound, and sold the same for 9d. per pound. What per cent. did I gain?
Ans. 12$\frac{1}{2}$ per cent.

(6.) Bought 650lb. of sugar at $0,10 a pound, and sold the same for $0,12 a pound. What was my gain per cent? Ans. 20 per cent.

(7.) If I buy tea at $1 a lb. and sell it again at $0,875 a lb., what is lost per cent.? Ans. 12$\frac{1}{2}$ per cent.

(8.) If 1 pound of tobacco cost $0,16, and be sold for $0,20; what will be the gain per cent.?
Ans. 25 per cent.

(9.) If 1cwt. of iron cost $3,43, what must it be sold for to gain 15 per cent.? Ans. $3,944.

(10.) Bought shalloon at $0,40 a yard, and sold it at 12$\frac{1}{2}$ per cent. loss. How did I sell it per yard?
Ans. $0,35.

(11.) If I buy Irish linen at 2s. 3d. per yard, how must I sell it per yard to gain 25 per cent.? Ans. 2s. 9$\frac{3}{4}$d.

(12.) If tea cost $0,54 per lb., how must it be sold per lb. to lose 12$\frac{1}{2}$ per cent.? Ans. $0,472+.

(13.) Bought cloth at 17s. 6d. per yard, which not proving so good as I expected, I am obliged to lose 15 per cent.; how must I sell it per yard? Ans. 14s. 10$\frac{1}{2}$d.

(14.) If I buy corn for $0,80 a bushel, how must I sell it in order to lose 15 per cent.? Ans. $0,68.

§ 145. 1. It is often required, when money is due at several periods of time, to *equate the time of the several payments*, so that the money may be paid and received at one time without loss to either party.

2. The times, in such cases, are first brought to the same unit, by multiplication of each payment by its time, in the lowest denomination given.

The amount embraced in this unit of time, is then obtained, and is the dividend; while the amount of the payments is the divisor.

3. *Illustration.*

If A owes B $380, to be paid as follows: $100 in 6 months, $120 in 7 months, and $160 in 10 months; what is the equated time for the payment of the whole debt?

$	mo.	$	
100 ×	6 =	600	for 1 month, the unit of time.
120 ×	7 =	840	do.
160 ×	10 =	1600	do.
380		3040	
		8 months.	

4. *Examples.*

(1.) A merchant owes $600, of which $100 is to be paid in 4 months, $200 in 10 months, and the remainder in 16 months; if he pays the whole at once, at what time must he make the payment? Ans. 12 months.

(2.) A merchant owes $600 to be paid in 12 months, $800 to be paid in 6 months, and $900 to be paid in 9 months; what is the equated time of payment?

Ans. 8 months, $22\frac{4}{23}$ days.

(3.) A owes B $600; $\frac{1}{3}$ of which is to be paid in 6 months, $\frac{1}{4}$ in 8 months, and the remainder in 12 months; what is the mean time of payment? Ans. 9 months.

(4.) A merchant has due him $1500; $\frac{1}{6}$ is to be paid in 2 months, $\frac{1}{3}$ in 3 months, and the rest in 6 months; what is the equated time for the payment of the whole?

Ans. $4\frac{1}{3}$ months.

(5.) The sum of $600 is to be paid in the following manner: $\frac{1}{4}$ in 6 months, $\frac{1}{3}$ in 8 months, and the remainder

in 10 months; what is the equated time for the payment of the whole? Ans. $7\frac{1}{3}$ months.

(6.) B owes C $190, to be paid as follows: $50 in 6 months, $60 in 7 months, and $80 in 10 months; what is the equated time to pay the whole? Ans. 8 months.

(7.) A merchant has owing him £300, to be paid as follows: £50 in 2 months, £100 in 5 months, and the rest in 8 months; what is the equated time to pay the whole? Ans. 6 months.

(8.) A merchant purchased goods to the amount of $2000, of which $400 are to be paid down, $800 in 5 months, and the rest in 10 months; but they agree to make one payment of the whole; what is the equated time? Ans. 6 months.

(9.) If $750 are to be paid, $\frac{2}{5}$ of it in $1\frac{1}{2}$ years, $\frac{3}{10}$ of it in 2 years, and the residue in $2\frac{1}{2}$ years, what is the equated time for paying the whole? Ans. $23\frac{2}{5}$ months.

§ 146. 1. In Partnerships it is often required to determine the shares of the profits or losses of the respective partners.

2. *Illustration.*

Three persons formed a partnership: A put in $1212; B $4848; C $2424. Within a certain time they gained $800. What was each partner's share of the gain?

This question is practically: If $1212+$4848+$2424; that is, $8484, give $840 profit, what does each of those sums separately give?

		profit.
		$\cancel{\$840}$
	$\cancel{\$8484}$	1212
$1212=A's.	707	70
$4848=B's.		
$2424=C's.	707	84840(120, A's profit.
		707
$8484=Whole Stock.		
$840=Whole Profit.		1414
		1414
		0

$$\begin{array}{r|l} & \text{profit.} \\ & \cancel{840} \\ \cancel{8484} & 4848 \\ 707 & 70 \\ \hline 707 & 339360(480, \text{ B's profit.} \\ & 2828 \\ \cline{2-2} & \;\;5656 \\ & \;\;5656 \\ \cline{2-2} & \;\;\;\;\;0 \end{array} \qquad \begin{array}{r|l} & \text{profit.} \\ & \cancel{840} \\ \cancel{8484} & 2424 \\ 707 & 70 \\ \hline 708 & 169680(240, \text{ C's profit.} \\ & 1414 \\ \cline{2-2} & \;\;2828 \\ & \;\;2828 \\ \cline{2-2} & \;\;\;\;\;0 \end{array}$$

Or, the ratio between the profit and the whole stock being once obtained, each man's stock may be at once multiplied by it, and so the operation be shortened. Thus;

$$\$8484: \ 840 = \tfrac{840}{8484} = \begin{array}{r|l} \cancel{8484} & \cancel{840} \\ 707 & 70 \end{array}$$

Then the several shares would be:

$$\text{A's} \quad \begin{array}{r|l} & 70 \\ 707 & 1212 \end{array} \qquad \text{B's} \quad \begin{array}{r|l} & 70 \\ 707 & 4848 \end{array} \qquad \text{C's} \quad \begin{array}{r|l} & 70 \\ 707 & 2424 \end{array}$$

3. *Examples.*

(1.) A and B made a joint stock of \$500; of which A put in \$350, and B \$150; they gain \$75; what is each man's share of the gain? Ans. { A's \$52,50. B's \$22,50.

(2.) A, B and C companied; A put £480, B £680, C £840, and they gained £1010; what is each man's share?
Ans. A's £242 8s.; B's £343 8s.; C's £424 4s.

(3.) Divide \$160 among 4 men so that their shares shall be as 1, 2, 3, and 4. Ans. \$16; \$32; \$48; \$64.

(4.) Three merchants trading together, gained \$800. A's stock was \$1200, B's \$4800, and C's \$2000; what was each man's share of the gain?
Ans. A's share \$120; B's \$480; and C's \$200.

(5.) D, E, and F, trading together, gained £120; D's stock was £140, E's £300, and F's £160; what was each man's share of the gain?
Ans. D's share was £28; E's £60; F's £32.

(6.) A, B, and C freighted a ship with 108 tuns of wine; of which A had 48 tuns, B 36, and C 24; by reason of stormy weather they were obliged to cast 45 tuns overboard; how much of the loss must each man sustain?

Ans. A 20 tuns; B 15; and C. 10.

(7.) A and B buy certain merchandize amounting to £160; of which A pays £90, and B £70; they gain by the purchase £32; what is each one's share of the profits?

Ans. A's share £18; B's £14.

(8.) A and B have a joint stock of $2100; of which A owns $1800 and B $300; they gain in a year $1000; what is each man's share of the gain?

Ans. A's $857,14+; B's $142,85+.

§ 147. 1. Often the stock of the respective partners is continued in use through unequal times. In such cases, the stock of each is multiplied by his time, so obtaining the same unit of time for each (§ 145. 2), and then the gain or loss of each is determined as before.

2. *Illustration.*

Three merchants formed a partnership: A put in $126 for 9 months; B $175 for 7 months; C $266 for 4 months. They gained $189. What was each man's share of the gain?

	$	mo.	$	
A	126	×9	=1134	for 1 month.
B	175	×7	=1225	do.
C	266	×4	=1064	do.

$3423=Whole amount for 1 month.
$189=Whole profit.

Then,
$$\begin{array}{r|l} \cancel{3423} & \cancel{189} \\ \cancel{489} & \cancel{27} \\ 163 & 9 \\ \hline 163 & 9 \end{array}$$

$$\begin{array}{r|l} & \text{profit.} \\ & 9 \\ 163 & 1134 \\ \hline 163 & 10206(62\tfrac{100}{163}, \text{ A's share.} \\ & \underline{978} \\ & 426 \\ & \underline{326} \\ & 100 \end{array}$$

```
    | profit.                              | profit.
    | 9                                    | 9
163 | 1225                             163 | 1064
--- | ----                             --- | ----
163 | 11025(67 104/163, B's share.     163 | 9576(58 122/163, C's share.
      978                                    815
      ----                                   ----
      1245                                   1426
      1141                                   1304
      ----                                   ----
       104                                    122
```

3. *Examples.*

(1.) A and B enter into partnership; A puts in £840 for 4 months, and B puts in £650 for 6 months; they gain £300, what is each man's share? Ans. A's £138 16s. 10d.
B's £161 3s. 1d.

(2.) A put in trade £50 for 4 months, and B £60 for 5 months; they gained £24, how is it to be divided between them? Ans. A's share £9 12s.; B's £14 8s.

(3.) C. and D hold a pasture together, for which they pay £54; C pastures 23 horses for 27 days, and D 21 horses for 39 days; how much of the rent ought each one to pay?
Ans. C £23 5s. 9d.; D £30 14s 3d.

(4.) Three merchants traded together; A put in £120 for 9 months, B £100 for 16 months, and C £100 for 14 months; they gained £100; what is each man's share?

Ans. A's 26 9s. 4¾+d.
B's £39 4s. 3¾+d.
C's £34 6s. 3¼+d.

(5.) Three merchants traded together, with a capital of $2300, of which A put in $620 for 8 months; B $950 for 11 months; and C. $730 for 13 months; and they gained $1800; what was each man's share?

Ans. A's $358,554+.
B's $755,421+.
C's $686,024+.

(6.) Three persons had received $655 interest; A had put in $4000 for 12 months, B $3000 for 15 months, and C $5000 for 8 months; what was each man's share of the interest? Ans. A's $240; B's $225; C's $200.

(7.) Two merchants traded in company; A put in $215

for 6 months, and B $390 for 9 months, but by misfortune they lost $200; how must they share the loss?

Ans. A $53,75; B $146,25.

§ 148. 1. The mean value of several things of the same kind having different values, is readily ascertained by the principles now possessed. The operation is usually termed *Alligation;* but is in fact ratio simply, and corresponds with the application lately made of it in finding the mean time for the payment of moneys due at different times.

2. The mode for finding the mean value of several things of different values, may be stated thus;

Find the cost of the whole quantities given, by multiplication of each by its price, and addition of the products; and divide the sum by the whole quantity.

3. *Illustration.*

A wine merchant bought several kinds of wine, as follows; 130 bottles at 40 cents each; 70 at 15 cents; 240 at 30 cents; 35 at 50 cents. He afterwards mixed the wine together, and again bottled it. Now what was the cost to him of a bottle of the mixture?

130	at	40=	5200	cents
70	"	15=	1050	"
240	"	30=	7200	"
35	"	50=	1750	"
475	cost		15200	cts.=32 cts. each bottle.

§ 149. 1. It is sometimes required to find what quantity of two different ingredients will make a mixture of a certain kind.

2. In such cases, if the value of the mixture required exceeds that of one of the ingredients just as much as it falls short of that of the other, it is evident that equal quantities of each will make the required compound.

Thus, if it be required to mix wine worth $2 per gallon with that worth $3, so that the mixture may be worth $2,50 per gallon, equal quantities of each must be taken, or quantities in the ratio of 1 to 1.

3. So if the value of the mixture exceeds that of one of the ingredients twice as much as it falls short of that of the other, their quantities would be two of one, the less, to one of the other, or their ratio as one half to one; $\frac{1}{2}$: 1.

Thus, if it be required to mix wine worth \$2 per gallon with that worth \$3, so that the mixture may be worth \$2,66⅔, the quantities would be two of the less, to one of the other, or in the ratio of ½ to 1; or of $\frac{1}{66\frac{2}{3}}$ to $\frac{1}{33\frac{1}{3}}$; the price of the mixture exceeding one of the ingredients just twice as much as it falls short of that of the other.

Generally, therefore, the nearer the value of the mixture is to that of one of the ingredients, the greater must be the quantity of this ingredient with respect to the other; and the reverse.

5. Hence, the mode of finding the proportion of two ingredients of a given value necessary to make a compound of a required value, is as follows:

Make the difference between the value of each ingredient and that of the compound, the denominator of a fraction whose numerator is one.

The fractions thus obtained will express the proportion required; and, reduced to a common denominator, the numerators will express the same proportion, or show what quantity of each ingredient is to be taken to make the required compound.

6. *Illustration.*

Mix wine at 5s. per gallon with that at 8s. per gallon, so that the mixture may be worth 6s. per gallon.

8s.—6s.=2s. over the price of the compound.
6s.—5s.=1s. under do. do.

Therefore ½ to ½=2 to 1, is the ratio of the required quantities.

Then, 1 gallon at 8s.=8s. and
2 gallons at 5s.=10s. make the mixture.
And, 3 gallons cost 18s.=6s. per gallon, the required price.

§ 150. 1. When the compound is limited to a certain quantity, the proportion of the ingredients corresponding to it is easily found by comparison. The order of comparison is as follows:

Make the whole quantity, found as above, the denominator of a fraction, of which the numerator is the quantity required; and compare them with reference to each part, obtained as above.

2. *Illustration.*

Mix wine at 5s. per gallon with that at 8s. per gallon, so that there may be 60 gallons, worth 6s. per gallon.

8s.—6s.=2s.
6s.—5s.=1s.

Therefore the ratio is $\frac{1}{2}$ to 1, or 2 to 1. And the whole quantity is 2+1=3.

The quantity required is 60 gallons; therefore the comparisons are,

$\frac{60}{3}$ of $2=\frac{120}{3}=40$; and
$\frac{60}{3}$ of $1=\frac{60}{3}=20$; and
40+20=60, the quantity required.
60 gallons at 6s.=360s. 40 at 8s.+20 at 6s.=360s.

§ 151. 1. When one of the ingredients is limited, the result is obtained just as easily. The comparisons are, as follows:

Make the quantity of the ingredient which is not limited, found as above, the denominator of a fraction of which the required quantity is the numerator; and, the quantity of the other ingredient that with reference to which comparison is made.

2. *Illustration.*

How many gallons of water at 0s. per gallon, should be mixed with 30 gallons of wine at 6s. per gallon, so that the compound may be worth 5s. per gallon?

5s.—0s.=5s.
6s.—5s.=1s.

Therefore the ratio of the quantities will be as $\frac{1}{5}$ to 1, or as 1 to 5.

The quantity of the ingredient not limited is 5.

The quantity required is 30.

Therefore the comparison is $\frac{30}{5}$ of $0=\frac{30}{5}=6$, the number of gallons required.

30 gallons at 6s.=180s.=36 gallons at 5s.=180s.

§ 152. 1. As we have found the proportion of two ingredients necessary to form a compound of a required value, so also we may consider either of these in connec-

tion with a third, with a fourth, and so on, thus making a compound of any required value, consisting of any number whatever of simple ingredients. The two ingredients used, however, must always be, one of a greater and the other of a less value, than that of the compound required.

2. *Illustration.*

A grocer would mix teas at 12s. and 10s. per lb., with 40lb. at 4s. per lb., so that the compound may be worth 8s. per lb. What quantities of each must be taken?

1st. We compare the tea at 4s. with that at 10s.

8s.—4s.=4s.
10s.—8s.=2s.

Hence, the ratio of the quantities at 4s. and 10s. is as $\frac{1}{4}$ to $\frac{1}{2}$, or, $\frac{1}{2}$ equalling $\frac{2}{4}$, as 1 to 2. Consequently, the proportion of the ingredients is 1lb. at 4s. to 2lb. at 10s.

2d. We compare the tea at 4s. with that at 12s.

8s.—4s.=4s.
12s.—8s.=4s.

Hence the ratio of the quantities at 4s. and 12s. is as $\frac{1}{4}$ to $\frac{1}{4}$, or as 1 to 1. Consequently the proportion of the ingredients is 1lb. at 4s. to 1lb. at 12s.

3d. We add together the proportions of each ingredient taken, thus obtaining the amount of each, and then by comparison, obtain the proportional part of each in the whole mixture.

We have then, in the first comparison above, 1 at 4s., and in the second 1 at 4s., making 2 at 4s.; and we have 2 at 10s., and 1 at 12s.

Thus the several quantities are as 2, 2, 1.

Therefore, the comparison,

$$\frac{40}{2} \text{ of } 2 = \frac{80}{2} = \quad 40 \text{ at } \ 4\text{s.} = 160\text{s.}$$
$$\frac{40}{2} \text{ of } 2 = \frac{80}{2} = \quad 40 \text{ at } 10\text{s.} = 400\text{s.}$$
$$\frac{40}{2} \text{ of } 1 = \frac{40}{2} = \quad 20 \text{ at } 12\text{s.} = 240\text{s.}$$

100lb. cost 800s. = 8s. per lb.

§ 153. *Examples.*

(1.) If 20 bushels of wheat at $1,35 per bushel be mixed with 10 bushels of rye at $0,90 per bushel, what will a bushel of this mixture be worth? Ans. $1,20.

(2.) A tobacconist mixed 36lb. of tobacco at 1s. 6d. per lb., 12lb. at 2s. a pound, with 12lb. at 1s. 10d. per lb.; what is the price of a pound of this mixture?
Ans. 1s. 8d.

(3.) A grocer mixed 2 cwt. of sugar at 56s. per cwt., and 1 cwt. at 43s. per cwt., and 2 cwt. at 50s. per cwt. together; what is 3 cwt. of the mixture worth?
Ans. £7 13s.

(4.) Suppose 5lb. of gold of 22 carats* fine, 2lb. of 21 carats fine, and 1lb. of alloy to be melted together, what is the quality of this mass? Ans. 19 carats fine.

(5.) A goldsmith melted together 5lb. of silver bullion of 8oz. fine, 10lb. of 7oz. fine, and 15lb. of 6oz. fine; what is the quality of this composition?
Ans. 6oz. 13pwt. 8gr. fine.

(6.) If 4oz. of silver worth $0,75 an ounce, be melted with 8oz. worth $0,60 an ounce, what will an ounce of the mixture be worth? Ans. $0,65.

(7.) If with 40 bushels of corn at 4s. per bushel, there are mixed 10 bushels at 6s. per bushel, 30 bushels at 5s. per bushel, and 20 bushels at 3s. per bushel, what will be the value of 5 bushels of this mixture?
Ans. £1 1s. 6d.

(8.) A grocer mixed 20lb. of raisins at $0,08 per lb., with 30lb. at $0,10, and 40lb. at $0,12; what is 1 pound of this mixture worth? Ans. $0,10⅘.

(9.) A merchant would mix wines worth 16s., 18s., and 22s. per gallon in such a way that the mixture be worth 20s. per gallon; how much of each sort must be taken?
Ans. 2 gal. at 16s.; 2 at 18s.; and 6 at 22s.

(10.) How much gold, of 15, of 17, and of 22 carats fine, must be mixed with 5oz. of 18 carats fine, so that the composition may be 20 carats fine?
Ans. 5oz. of 15 carats fine; 2 of 17; and 25 of 22.

* A carat is the twenty-fourth part; 22 carats fine means $\frac{22}{24}$ of pure metal. A carat is also divided into four parts, called grains of a carat.

(11.) A goldsmith has gold of 16, of 18, of 23, and of 24 carats fine; what part of each must be taken so that the mixture shall be 21 carats fine?

Ans. 3 of 16; 2 of 18; 3 of 23; and 5 of 24.

(12.) What proportions of coffee at 16¢, 20¢, and 28¢ per lb. must be mixed together so that the compound shall be worth 24¢ per lb.?

Ans. In the proportion of 4lb. at 16¢; 4lb. at 20¢; and 12lb. at 28¢.

(13.) What portion of brandy at 14s. per gallon, of old Madeira at 24s. per gallon, of new Madeira at 21s. per gallon, and of brandy at 10s. per gallon, must be mixed together so that the mixture shall be worth 18s. per gallon?

Ans. 6 gal. at 10s.; 3 at 14s.; 4 at 21s.; and 8 at 24s.

(14.) A grocer having four sorts of tea worth 5s., 6s., 8s., and 9s. per lb. wishes a mixture of 87lb. worth 7s. per lb.; how much of each sort must be taken?

Ans. 29lb. at 5s.; 14½lb. at 6s.; 14½ at 8s.; and 29lb. at 9s.

(15.) A vintner has 4 sorts of wine; white wine at 4s. per gal., Flemish at 6s. per gal., Malaga at 8s. per gal., and Canary at 10s. per. gal.; he would make a mixture of 60 gallons to be worth 5s. per gallon; what quantity must be taken of each?

Ans. 45 gal. of white wine; 5 gal. of Flemish; 5 gal. of Malaga; and 5 gal. of Canary.

(16.) A silversmith has 4 sorts of gold; of 24, of 22, of 20, and of 15 carats fine; he would make a mixture of 42oz. of 17 carats fine; how much must be taken of each sort?

Ans. 4 of 24; 4 of 22; 4 of 20; and 30 of 15 carats fine.

(17.) How much wine at 5s., at 5s. 6d., and 6s. a gallon must be mixed with 4 gallons at 4s. a gallon so that the mixture shall be worth 5s. 4d. per gallon?

Ans. 1 gal. at 5s.; 2 at 5s. 6d.; and 8 at 6s.

(18.) A farmer would mix 14 bushels of wheat at $1,25 a bushel, with rye at $0,72, barley at $0,48, and oats at $0,36; how much must be taken of each sort to make the mixture worth $0,64 per bushel?

Ans. 14 bu. of wheat; 8 bu. of rye; 4 bu. of barley; 28 bu. of oats.

(19.) There is a mixture made of wheat at 4s. per bushel, rye at 3s., barley at 2s., with 12 bushels of oats at 18d. per bushel; how much is taken of each sort when the mixture is worth 3s. 6d.?

Ans. 96bu. of wheat; 12bu. of rye; 12bu. of barley; and 12bu. of oats.

(20.) A distiller would mix 40 gal. of French brandy at 12s. per gallon, with English at 7s. and spirits at 4s. per gallon; what quantity must be taken of each sort, that the mixture may be afforded at 8s. per gallon?

Ans. 40 gal. French; 32 gal. English; and 32 gal. of spirits.

(21.) If 8 bushels of rye at 50 cents a bushel, be mixed with 12 bushels of corn worth 65 cents a bushel, and 6 bushels of oats at 30 cents a bushel; what is a bushel of the mixture worth? Ans. \$0,523.

(22.) Suppose that a number of men work at a certain work during a month, as follows, viz.: 6 men work 15 days each; 4 men work 19 days each; 12 men work 20 days each; and 10 men work 26 days each, during that time; on how many days' work, on an average, can one calculate for each man, in a month? Ans. $20\frac{13}{16}$ days.

(23.) A goldsmith having gold 15 carats fine, 19 carats, 21 carats, and 24 carats, wishes to make a mixture 20 carats fine; how much of each must he take?

Ans. 440 carats.

(24.) If a grocer have sugars worth 11 cents, 13 cents, 14 cents, 15 cents, and 16 cents a pound; in what proportions must he mix them, in order that the mixture may be worth 12 cents per pound?

Ans. 10lb. at 11¢, and 1lb. of each of the others.

(25.) What quantity of rye at 48 cents, of corn at 36 cts., and of barley at 30 cents a bushel, being mixed with 10 bushels of wheat worth 70 cents a bushel, will form a mixture worth 38 cents a bushel?

$$\text{Ans.}\begin{cases} 2\frac{1}{2} \text{ bushels of rye.} \\ 12\frac{1}{2} \text{ do. corn.} \\ 40 \text{ do. barley.}\end{cases}$$

(26.) A farmer mixed 15 bushels of rye, at 64 cents a bushel; 18 bushels of corn, at 55 cents a bushels; and 21 bushels of oats at 28 cents a bushel; what is a bushel of the mixture worth? Ans. \$0,47.

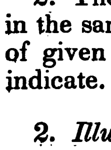
PART V.

POWERS AND ROOTS; OR, INVOLUTION AND EVOLUTION.

I. INVOLUTION.

§ 154. 1. When two numbers which are equal are multiplied together, or a number is multiplied by itself, the product is surface (§ 114. 5); and is called a *square*, because it corresponds to what would be produced in nature by laying off the quantity which these numbers represent in any unit of lineal measure, in two directions perpendicular to each other, and completing the figure by two equal lines drawn perpendicular at the end of them.

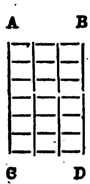
2. *Illustration.*



Taking A B=4ft., and A C=4ft., and then drawing B D and C D perpendicular to A B and A C, and the square A B C D is the result; representing the square of 4ft.; that is, 4ft.×4ft.=16ft.; and contains 16 square feet.

2. The product of any two numbers may be represented in the same way, by two lines perpendicular to each other, of given dimensions, and completing the figure which they indicate.

2. *Illustration.*



Taking A B=3ft., and A C=6ft., and completing the figure, as above, by drawing B D and C D, and the *rectangular* figure A B C D is the result; representing the product of 3ft.×6ft.=18ft.; and contains 18 square feet.

3. It is evident then that when we have a surface of either of the kinds now considered and one of the sides given, the other side may be obtained by division, for in each case we have a product and one factor given to obtain the other factor (§ 36. 4). In the former case, indeed, the giving of one side gives the other, the two sides being equal, so that division to obtain it is not necessary.

4. But, when the figure is a *square* and the surface only is given, and the sides are required, a peculiar mode of operation is resorted to, which is called *Extraction of the Square Root.*

§ 155. 1. The multiplication of a number into itself, is called *Involution;* the result or product of such multiplication is called a *power;* and the number itself, a *root.*

2. The number of times a root is embraced in a product or power, numbers that power. Thus, if a number be multiplied by itself, the product is called the *second* power, or square; as $4\times4=16$, the *second* power or *square* of 4. If this power be again multiplied by the root, the product is called the *third* power, or *cube;* as $4\times4\times4=64$, the *third* power, or *cube* of 4. If the third power be multiplied by the root, the product is the *fourth* power, or *bi-quadrate;* and so on.

3. In Arithmetic, the power to which a number is required to be raised is denoted by a figure written at the right, and a little above the number, which is called the *index*, or *exponent* of that power. Thus, if we involve 2, we have,

$2=2$, the *root.*
$2\times2=2^2$, the *second* power, or square of 2.
$2\times2\times2=2^3$, the *third* power, or cube of 2.
$2\times2\times2\times2=2^4$, the *fourth* power, or bi-quadrate of 2.
$2\times2\times2\times2\times2=2^5$, the *fifth* power of 2.
$2\times2\times2\times2\times2\times2=2^6$, the *sixth* power of 2.

The *exponent* then shows how many times the root is to be a factor in the power required, and is always 1 greater than the number of multiplications producing the power.

This use of the exponent differs from its use in algebra, where it is annexed to the power, and shows how many times the root is a factor in it.

4. The units, tens, &c., of any number may be involved separately; and their *sum* will equal the required power.

The only use of this mode of involution is to show distinctly what a power is composed of.

Illustrations.

1st. $14=10+4$; and $14^2=14\times14=10+4\times10+4=196$.

$$\begin{array}{r} 10+4 \\ 10+4 \\ \hline 4\times10+4\times4 \\ 10\times10+4\times10 \quad\quad \\ \hline \end{array}$$

$10\times10+2\times4\times10+4\times4=10^2+2\times4\times10+4^2=100+80+16=196$.

Thus, we obtain as the product of the units of the multiplier into the units of the multiplicand, $4\times4=16$; as the product of the units of the multiplier into the tens of the multiplicand, $4\times10=40$; as the product of the tens of the multiplier into the units of the multiplicand, $10\times4=40$; and as the product of the tens of the multiplier into the tens of the multiplicand, $10\times10=100$.

We see then, that the *second* power, or *square* of 14, equals; 1st, the product of the first part into itself, 10×10; 2d, twice the product of the two parts into each other, $2\times4\times10$; 3d, the product of the second part into itself, 4×4.

2d. So, $14^3=14\times14\times14=10+4\times10+4\times10+4=2744$.

$10+4\times10+4=10\times10+2\times10\times4+4\times4$, which being multiplied by $10+4$ gives

$$\begin{array}{l} 10\times10\times10+2\times10\times4\times10+4\times4\times10 \\ \quad\quad +4\times10\times10 \quad +2\times4\times4\times10+4\times4\times4 \\ \hline \end{array}$$

$10\times10\times10+3\times10\times10\times4+3\times10\times4\times4+4\times4\times4=10^3+3\times10^2\times4+3\times10\times4^2+4^3=1000+1200+480+64=2744$.

It will be observed, that this product is composed of the cube of 10; three times the product of the square of 10 into 4; three times the product of 10 into the square of 4; and the cube of 4; or, generally, the cube of the first part; three times the product of the square of the first part into the second; three times the product of the first into the

square of the second part; and the cube of the second part.

5. The second power or square of a number cannot have more figures than double those in the root; and at least, but one less; the third power or cube cannot have more than triple the figures in the root, and at least, but two less; and so on. The pupil may illustrate this for himself.

This fact enables us to decide at once the number of figures composing the root of any given power. Thus, if we have the second power of one or two figures, we know the root to consist of one; if the power consist of three or four figures, we know the root to have two; if of five or six, the root must have three, and so on.

The first figure in every product or power, as it is obtained by multiplication, falls at the right, in the place of units, and the others at its left.

6. Knowing then, the number of figures of which the root of any given power may consist, we may begin at the right of a power and separate it into as many parts as there are figures in its root.

Thus may be indicated the number of figures composing the root. This is usually done by means of the point or period; writing it first over the units place, and then over every second, third, fourth, &c., figure thereafter, according as the power is the second, third, fourth, &c. These points mark where the right hand figure in the product of each figure in the root falls; unless, indeed, the number be decimals, in which case the pointing is in the same order towards the right; and also show of how many figures the root consists.

7. If a vulgar fraction is to be involved, each term is separately multiplied; if a mixed number, it is first changed to an improper fraction or a decimal.

8. If any number be raised to two different powers, the power which is obtained by multiplying these two powers together, is expressed by adding their indices, thus: $2^2 \times 2^3 = 2^5 = 32$; for $2^2 = 4$, and $2^3 = 8$, and $8 \times 4 = 32$; and $2 \times 2 \times 2 \times 2 \times 2 = 32$.

Or, again, any power of a given number is divided by another power of the same number by subtracting the index of the divisor from the index of the dividend, thus: $2^5 \div 2^3 = 2^2$; for $2^5 = 32$, and $2^3 = 8$, and $32 \div 8 = 4$, the second power of 2.

II. EVOLUTION.

§ 156. 1. EVOLUTION is the obtaining of the root to a given power; and is therefore often called *Extraction of the root*. It is the reverse of involution.

2. It is indicated by the sign $\sqrt{}$, or an extended r, termed the radical sign, with the number of the power written in the angle at its left. Thus $\sqrt{196}$, or, for this root, $\sqrt[2]{196}$, denotes the square root of 196; $\sqrt[3]{2744}$, denotes the third or cube root of 2744, and so on. Or, another method is to continue the notation of powers used in involution, expressing the roots in their corresponding fractions. Thus, $\sqrt{196}=(196)^{\frac{1}{2}}$; $\sqrt[3]{2744}=(2744)^{\frac{1}{3}}$; and so on.

3. It is obvious that although there is no number of which there may not be obtained an exact power, there are many of which precise roots can never be determined; because every number is not the product of another number multiplied by itself; in the same manner as in division every number is not divisible by a given number.

By decimals however, we can approximate towards the root to any required degree of exactness.

Roots which approximate are called *surd* roots; those which are exact *rational* roots.

§ 157. 1. SQUARE ROOT. The first step in the extraction of the square root, the figures of the power being divided off, from the right, in pairs by points, is to find the number which multiplied into itself will give the product nearest the left hand subdivision of the given power; which number is the first or highest figure of the required root.

This square being subtracted from the whole power, the remainder must furnish the two other products; that is, the product of twice the first number of the root into the second, and that of the second into itself (§ 155. 6, 1st.).

The next step is to multiply the first number found by 2; divide the remainder of the given power by the product,

to obtain the second number of the root; unite this number by addition to the divisor—the product of twice the first—and then multiply the sum by it.

This gives the last two products required; that is, twice the first by the second, and the second by itself.

The power is then absorbed entirely, if it have a *rational* root; if it have not, the root is *surd*, and the exact root is approximated, by continuing the same mode of operation by decimals.

2. In the determination of the second quotient figure, it must be observed, that there must be left a sufficient latitude for the subtraction of not only its double product with the first number found, but also of the square of this quotient.

3. A reference to involution will show how these steps are deduced, and why they are necessary (§ 155. 6).

4. *Illustrations.*

1st. Let $\sqrt{196}$ be taken to illustrate the inversion of obtaining the power.

	$1\dot{9}\dot{6}(10+4=14$	
First square $10^2=10\times10=$	100	Or, $1\dot{9}\dot{6}(14$
	——	1
Divisor, $2\times10=20$.	96 quotient 4	——
$20+4=24$. $24\times4=$	96	$24)96$
	—	96
	00	

(1.) The pointing off of the numbers shows that the root has two places of figures. The first figure, therefore, will be in the tens. And the number in the left hand subdivision of the power being 1, the square root of which is also 1, the first square will be $10\times10=100$.

Subtracting this square, from the whole 196, leaves as remainder 96.

(2.) The divisor which is to find the other number of the root, is the product of twice the first number; $2\times10=20$; which being found to go 4 times in the remainder, 4 is written in the root, and also added to the 20; $20+4=24$; this sum is then multiplied by 4, as we have found it to be a factor in the last two terms; $24\times4=96$.

This sum written under the remainder 96, and subtracted, leaves no remainder. Therefore the quotient 14, is the exact square root of 196.

2d. Let the square root of 13456 be extracted.

```
                               13456(100+10+6=116.
       100²=100×100=10000
                               -----
Divisor, 2×100=200.            3456  quotient=10.
  200+10=210.   210×10=2100
                               -----
Divisor, 2×110=220.            1356  quotient=6.
    220+6=226.   226×6=1356
                               -----
                               0000
```

```
Or, 13456(116
     1
    ---
21) 34
    21
    ----
226)1356
    1356
    ----
    0000
```

3d. $\sqrt{1419,7824}$.

```
      1419,7824(37,68
      9
      ---
67)   519
      469
      ---
74,6) 50,78
      44,76
      -----
75,28) 6,0224
       6,0224
       ------
       00000
```

4th. $\sqrt{2}$.

```
      2)1,41421+
       1
       —
   24)1,00
        96
       ——
  281)  400
        281
        ———
 2824) 11900
       11296
       —————
28282)  60400
        56564
        —————
282841) 383600. &c.
```

5. For the convenience of the pupil, we may embrace this operation in a case and rule.

Case, *To extract the square root.*

Rule. The given power being distinguished into periods of two figures each, by a point over the units, another over the hundreds, and so on; or, if there be decimals, by pointing in the same manner towards the right:

I. Find the greatest root in the first or left hand period, and write it in the quotient; square it, and write the square under the period; subtract it from it; and to the remainder bring down the next period for a dividend.

II. Double the root for a divisor; find how many times this divisor, with the quotient sought annexed to it, is contained in the dividend; write the result in the quotient, and at the right of the divisor; multiply the whole divisor thus obtained by this quotient figure; write the product under the dividend; and subtract and bring down as before.

III. Proceed thus till the given power is exhausted, when if the root be not exact, annex ciphers and approximate by decimals.

6. If the square root of a vulgar fraction be required, it is extracted from each term separately; or if the fraction be surd, it is first changed to a decimal.

7. *Examples.*

(1.) What is the square root of 106929? Ans. 327.

(2.) What is the square root of 2268741?
Ans. 1506,23+.

(3.) What is the square root of 7596796?
Ans. 2756,22+.

(4.) What is the square root of 36372961?
Ans. 6031.

(5.) What is the square root of 22071204?
Ans. 4698.

(6.) What is the square root of 45? Ans. 6,078.

(7.) What is the square root of 2025? Ans. 45.

(8.) What is the square root of 17,3056? Ans. 4,16.

(9.) What is the square root of 566,44? Ans. 23,8.

(10.) What is the square root of 47,692836?
Ans. 6,906.

(11.) What is the square root of 4,426816?
Ans. 2,104.

(12.) What is the square root of ,000729? Ans. ,027.

(13.) What is the square root of ,002916? Ans. ,054.

(14.) What is the square root of ,00103041?
Ans. ,0321.

(15.) What is the square root of $\frac{25}{36}$? Ans. $\frac{5}{6}$.

(16.) What is the square root of $\frac{27}{147}$? Ans. $\frac{3}{7}$.

(17.) What is the square root of $\frac{2304}{5184}$? Ans. $\frac{2}{3}$.

(18.) What is the square root of $\frac{2704}{4225}$? Ans. $\frac{4}{5}$.

(19.) What is the square root of $\frac{9216}{12544}$? Ans. $\frac{6}{7}$.

(20.) What is the square root of $20\frac{1}{4}$? Ans. $4\frac{1}{2}$.

(21.) What is the square root of $\frac{9}{12}$? Ans. ,866025.

(22.) What is the square root of $\frac{478}{549}$?
Ans. ,93309+.

(23.) What is the square root of $\frac{357}{476}$?
Ans. ,86602+.

(24.) What is the square root of $\frac{275}{341}$?
Ans. ,89802+.

§ 158. 1. Cube Root. The evolution of the cube root, like that of the square root, is precisely the reverse of its involution to the third power. A reference to that is therefore necessary; and the principles which arise from the converse operation are thence easily deducible (§ 155).

2. As for the evolution of the square root, the figures in

the power were divided off in pairs from the right by the point; here, as before stated (§ 155. 5), every three figures from the right are pointed off.

§ 159. 1. The first step in the extraction of the cube root, is to find the largest root, which cubed, will come nearest to the left hand subdivision of the given power.

This is then cubed, and the cube written under the power. This cube is then subtracted, leaving a remainder, which must furnish the other products; that is, three times the product of the square of the first number into the second; three times the product of the first into the square of the second; and the cube of the second (§ 155).

2. The next step is to form the triple product of the square of the first number, with which, as a divisor, to obtain the second—always allowing for the other products, over and above the product of this divisor by the quotient figure taken—then by means of this second number to form the three products necessary, indicated in the involution, and just specified, obtain their sum, and subtract them from the remainder, or new dividend.

3. If the root be exact, the power is now absorbed, unless the root embrace more than two figures, which is always shown by the points; in which case the same operations are repeated till the power is exhausted.

4. If the root be not exact, the operation may be carried on by decimals, and the exact root be approximated; three places of decimals being in each case annexed as a period.

5. *Illustrations.*

1st. Let the cube root of 2744 be extracted.

$$\sqrt{\dot{2}74\dot{4}}(10+4=14=\text{root}.$$
$$\text{1st. cube}=10^3=1000$$
$$\text{Divisor}=3\times10^2=300 \qquad \overline{1744}\ \text{quotient}=4$$

$$\left.\begin{array}{lr}
\text{1. Triple the square of the 1st. by the 2d.}=3\times10^2\times4= & 1200\\
\text{2. " the first into the sq. of the 2d.}=3\times10\times4^2= & 480\\
\text{3. The cube of the second}= \quad 4^3= & 64\\
 & \overline{}\\
\text{Sum of the three products}= & 1744
\end{array}\right\}$$

$$1744$$
$$\overline{0000}$$

(1.) The only number which cubed will not exceed 2 is 1; which is shown to be a ten by the points. Taking away the cube of this, from the power, leaves as a remainder, or new dividend, 1744. Forming the triple products of the square of the first, $10^2=300$, for the divisor, we obtain the second number, or quotient 4.

In common division the quotient would be 5, but room being necessary for the subtraction of the three products, 4 only is taken.

(2.) Having thus the second number of the root, the three products are formed, as indicated; that is, triple the square of 10, triple 10 into 4^2, and 4^3, and their sum subtracted from the remainder 1744.

The power being thus exhausted, we know our root to be exact, and that it is 14.

2d. $\sqrt{99\dot{4}01\dot{1}99\dot{2}}(900+90+8=998.$

1st. cube $=900^3=729000000$

1st. divisor $=3\times900^2=2430000$ 265011992 quotient $=90$.

$$\left.\begin{array}{r} 3\times900^2\times90=218700000 \\ 3\times900\times90^2=21870000 \\ 90^3=729000 \\ \hline \text{Sum of three products}=241299000 \end{array}\right\}$$

241299000

2d. divisor $=3\times990^2=2940300$ 23712992 quotient $=8$.

$$\left.\begin{array}{r} \text{2d. term}=3\times990^2\times8=23522400 \\ \text{3d. term}=3\times990\times8^2=190080 \\ \text{4th. term}=\quad 8^3=512 \\ \hline \text{Sum of the three products}=23712992 \end{array}\right\}$$

23712992

00000000

(1.) The given power admitting three subdivisions, in beginning from the right, indicates a root of three places of figures.

The nearest cube root of the first subdivision on the left is 9, which being in the third place is equivalent to 900; the cube of it being obtained and subtracted, leaves the first remainder.

The triple product of the square of the first number of the root taken as a divisor shows 90 as a quotient for the next part of the root.

The products are now formed as indicated; and their sum obtained and subtracted from the remainder.

(2.) The same process takes place again as before, taking the whole of the root found, as the first time; and the sum of the products being equal to the last remainder, the power given proves an exact cube of the number 998, obtained as a root.

6. It is evidently possible to evolve roots of higher powers in the same manner; the principles for the operations being deduced from involution, and therefore the same for every class of powers.

The combinations of the factors in higher powers being, however, more complicated, the evolution of the roots are proportionably more difficult. Their treatment belongs rather to algebra than to arithmetic; therefore to that science they are left.

7. *Examples.*

(1.) What is the cube root of 250047 ? Ans. 63.
(2.) What is the cube root of 6859 ? Ans. 19.
(3.) What is the cube root of 205379 ? Ans. 59.
(4.) What is the cube root of 389017 ? Ans. 73.
(5.) What is the cube root of 5735339 ? Ans. 179.
(6.) What is the cube root of 32461759 ? Ans. 319.
(7.) What is the cube root of 84604519 ? Ans. 439.
(8.) What is the cube root of 259694072 ? Ans. 638.
(9.) What is the cube root of 48228544 ? Ans. 364.
(10.) What is the cube root of ,870983875 ? Ans. ,955.
(11.) What is the cube root of 162,771336 ? Ans. 5,46.
(12.) What is the cube root of ,000684134 ? Ans. ,088+.
(13.) What is the cube root of 2 ? Ans. 1,25+.
(14.) What is the cube root of $\frac{8}{64}$? Ans. $\frac{2}{4}$.
(15.) What is the cube root of $\frac{343}{729}$? Ans. $\frac{7}{9}$.
(16.) What is the cube root of $12\frac{19}{27}$? Ans. $2\frac{1}{3}$.
(17.) What is the cube root of $31\frac{15}{343}$? Ans. $3\frac{1}{7}$.

§ 160. *Miscellaneous Examples in Evolution.*

(1.) An army of 567009 men are drawn up in a solid body, in the form of a square; what is the number of men in rank and file? Ans. 753 men.

(2.) What is the length of the side of a square, which shall contain an acre, or 160 rods?

Ans. 12,649+ rods.

(3.) A certain square pavement contains 48841 square stones, all of the same size; how many are contained in one of the sides? Ans. 221 stones.

(4.) A section of land in the Western States is a square consisting of 640 acres; what is the length, in rods, of one of its sides? Ans. 320 rods.

(5.) What must be the side of a square field that shall contain an area equal to another field of a rectangular shape, the two adjacent sides of which are 18 rods and 72 rods? Ans. 36 rods.

(6.) What is the side of a square block of marble that contains 13824 cubic inches? Ans. 24 inches.

(7.) The custom house bushel contains 2150,4 cubic inches; what is the side of a cubic box that will contain 50 bushels? Ans 47,5 inches.

(8.) The contents of a cubical piece of timber is 103823 solid inches; how many inehes is it each way?

Ans. 47 inches.

(9.) A stone of a cubical form contains 474552 solid inches; what is the superficial contents of one of its sides?

Ans. 6084 inches.

(10.) If 484 trees be planted in a square orchard, how many must there be in a row each way?

Ans. 22 trees.

(11.) If the solid contents of a globe is 10648, what is the side of a cube of equal solidity? Ans. 22.

(12.) Suppose 3097600 men to be drawn up in a solid square; how many men would there be on a side? and allowing each man to occupy a square yard of ground, how large a plain would contain the whole number?

Ans. { 1760 men on a side.
1 mile square.

(13.) What is the superficial area of one of the faces of a cubical block containing 4096 solid feet? Ans. 256 sq. ft.

§ 161. *Miscellaneous Examples in all the Parts.*

(1.) How many pounds of butter at \$0,20 per pound, will pay for 14 yards of cloth at \$0,28 per yard?
Ans. 19,6 pounds.

(2.) How many bushels of corn at 5 shillings per bushel, must I give for 84 bushels of wheat at 7s. 6d. per bushel? Ans. 126 bushels.

(3.) How many cwt. of cheese at \$7 per cwt., must I give for 36 yards of broadcloth at \$5,50 per yard?
Ans. 28cwt. 1qr. 4lb.

(4.) How much tea at 5s. 6d. per pound, must be given for 32 gallons of brandy at 10s. 8d. per gallon?
Ans. 62,06 pounds.

(5.) A gives B 250 yards of cotton cloth, at \$0,30 per yard, for 324lb. of sugar; what was the sugar worth per pound? Ans. \$0,231+.

(6.) How much tea at 7s. 6d. per pound, must be given for 234 yards of flannel, at 3s. 9d. per yard?
Ans. 117 pounds.

(7.) How much wheat at \$1,25 per bushel, must be given for 50 bushels of rye at \$0,70 per bushel?
Ans. 28 bushels.

(8.) How much rice at 28s. per cwt., must be given for 3½ cwt. of raisins at 5d. per pound?
Ans. 5cwt. 3qr. 9⅓lb.

(9.) A had 8¼ cwt. of sugar at \$0,12 per pound, for which he gave B 18 cwt. of flour; what was the flour rated at per pound? Ans. \$0,055.

(10.) B delivered 3 hhds. of brandy at 6s. 8d. per gallon, to C, for 126 yards of cloth; what was the cloth per yard? Ans. 10s.

(11.) D gives E 250 yards of linen at \$0,30 per yard, for 319lb. of pepper; what was the pepper per pound?
Ans. \$0,2351.

(12.) What quantity of flax at \$0,09 per lb., must be given for 12lb. of indigo at \$2,19 per lb.? Ans. 292lb.

(13.) How many pounds of cinnamon at 10s. per lb., must be given for 5 cwt. of salæratus at 9d. per lb.?
Ans. 42lb.

(14.) What is the amount of \$400 for 2 years, at 6 per cent., compound interest? Ans. \$449,44.

(15.) What is the premium of insurance on $9870, at 14 per cent.? Ans. $1381,80.

(16.) What is the length of a pole, $\frac{1}{8}$ of which stands in the ground, 16 feet in the water, and $\frac{4}{5}$ in the air?

Ans. 213ft. 4in.

(17.) What is the premium on $1800, at 15 per cent.?

Ans. $270.

(18.) If 356 persons consume 75 barrels of provision in 9 months, how many barrels will 500 men consume in the same time? Ans. $105\frac{30}{89}$ barrels.

(19.) A can mow an acre of grass in $5\frac{1}{2}$ hours; B can mow 2 acres in 9 hours. In what time will they both mow $12\frac{1}{2}$ acres? Ans. $30\frac{15}{16}$ hours.

(20.) How much sugar at 9d. per lb. must be given in exchange for 492lb. of rice at 3d. per lb.? Ans. 164lb.

(21.) A young man received £350 as his share of his father's estate, which was $\frac{3}{5}$ of his elder brother's portion; and the elder brother's portion was $\frac{1}{4}$ of the whole estate. What was the whole estate? Ans. £2333 6s. 8d.

(22.) If $\frac{5}{7}$ of $\frac{1}{4}$ of $\frac{4}{5}$ of a ship be worth $\frac{2}{9}$ of $\frac{7}{8}$ of $\frac{12}{13}$ of her cargo, which was valued at $2400, what is the value of both ship and cargo? Ans. $5415,384+.

(23.) Bought a pipe of wine for $84, from which 12 gallons leaked out. What shall I gain, if I sell the remainder at $0,12$\frac{1}{2}$ a pint? Ans. $30.

(24.) What is the interest on $462,50 for 3 years, 6 months and 12 days, at 6 per cent.? Ans. $98,05.

(25.) How many square feet are there in a board 16 feet 8 inches long, and 9 inches broad?

Ans. $12\frac{1}{2}$ square feet.

(26.) There is a cistern having 2 pipes leading into it, one of which will fill it in 30 minutes, and the other in 45 minutes; in what time will both, running together, fill the same? Ans. 18 minutes.

(27.) What number added to the thirty-first part of 3813 will make the sum 200? Ans. 77.

(28.) A composition being made of 5lb. of tea, at 7s. per pound, 9lb. at 8s. 6d. per pound, and $14\frac{1}{2}$lb. at 5s. 10d. per pound, what is a pound of it worth?

Ans. 6s. $10\frac{1}{2}$d.

(29.) A canal contractor engaged to excavate 80 rods of earth in 15 days; after 30 men had been employed 6

days, 20 rods of the canal were completed. Does he require fewer or more men to complete the residue according to contract, and how many? Ans. 10 more men.

(30.) If $1600 were put at interest at 6 per cent. per annum, until it amounted to $2000, what would be the time? Ans. 4 years 2 months.

(31.) A man owes $800, due 4 years hence; what ought he to pay now, supposing money to be worth 5 per cent.? Ans. $666,66$\frac{2}{3}$.

(32.) A and B lay out certain sums in merchandize amounting to $320, of which A pays $180 and B $140; they gain by the purchase $64; what is each one's share? Ans. A's $36; B's $28.

(33.) From $\frac{3}{7}$ of $1\frac{1}{2}$ of 7, take $\frac{1}{4}$ of $\frac{3}{8}$. Ans. $4\frac{13}{32}$.

(34.) If 20 men can perform a piece of work in 12 days, how many men will accomplish another thrice as large, in a fifth part of the time? Ans. 300 men.

(35.) A and B can build a boat in 20 days; with the assistance of C they can do it in 12 days; in what time would C do it alone? Ans. 30 days.

(36.) In an orchard, $\frac{1}{2}$ the trees bear apples, $\frac{1}{4}$ pears, $\frac{1}{6}$ plums, and 50 of them cherries; how many trees are there in all? Ans. 600 trees.

(37.) A man received $130,40 interest for the use of $420, the rate being 6 per cent.; how long was the money at interest? Ans. 5 years 4 months.

(38.) If $\frac{1}{2}$ of a sum of money be now due, $\frac{1}{4}$ in 4 months, and the residue in 8 months, what is the equated time of payment? Ans. 3 months.

(39.) If 4 men, in $2\frac{1}{2}$ days, mow $6\frac{2}{3}$ acres of grass, by working $8\frac{1}{4}$ hours a day, how many acres will 15 men mow in $3\frac{3}{4}$ days by working 9 hours a day?

Ans. $40\frac{10}{11}$ acres.

(40.) The pedestal of a monument was a square block of granite, containing 373248 cubic inches; what was the length of one of its sides? Ans. 6 feet.

(41.) Three persons purchase a vessel in company, towards the payment of which A advanced $\frac{2}{5}$, B $\frac{3}{7}$, and C $900; what did A and B each pay, and what part of the vessel had C?

Ans. A paid $2100; B paid $2250; and C had $\frac{6}{35}$ of the vessel.

(42.) If 1cwt. cost £4 17s. 4d., what will 9cwt. 3qr. 14lb. cost at the same rate? Ans. £48 1s. 2d.

(43.) How many pounds, New England currency, are there in \$575,50? Ans. £172 12s.

(44.) At £3 10s. per cwt., what cost 1lb.? Ans. $7\frac{1}{2}$d.

(45.) How many gallons of molasses at \$0,08 per quart, must be given for 24cwt. of rice at \$8 per cwt.? Ans. 600 gallons.

(46.) Three merchants trading in company, suffered a loss of \$600; their several stocks were \$800, \$1000, and \$1200; what was each man's loss? Ans. \$160; \$200; \$240.

(47.) There is a room 20ft. square and 7ft. 6in. high, to be plastered, at 10d. New-York currency per square yard; how many dollars will it cost? Ans. \$6,94+.

(48.) What is the difference between the interest of £350 for 8 years, at 4 per cent., and the discount of the same sum, for the same time, at the same rate? Ans. £27 $3\frac{1}{33}$s.

(49.) Change $\frac{2\frac{1}{3}}{3\frac{4}{5}}$ to a single fraction. Ans. $\frac{35}{57}$.

(50.) What commission must I receive for selling \$478 worth of books, at 8 per cent.? Ans. \$38,24.

(51.) A man exchanges 760 gallons of molasses, at \$0,37½ per gallon, for 66½ cwt. of cheese, at \$4 per cwt.; how much will be the balance in his favor? Ans. \$19.

(52.) There is a square field containing 90 acres; how many rods in length is each of its sides? Ans. 120 rods.

(53.) A grocer mixed 10 cwt. of sugar at \$10 per cwt. with 8 cwt. at \$7½, and 4 cwt. at \$4 per cwt.; what is the price of the mixture per cwt.? Ans. \$8.

(54.) What sum in ready money will discharge a debt of \$552,50, due 5 years hence, at 6 per cent.? Ans. \$425.

(55.) Bought a pieee of broadcloth for \$375, and sold it for \$420. How much did I gain per cent.? Ans. 12 per cent.

(56.) What is the length of one side of a square tract of land which contains 102400 square rods? Ans. 320 rods.

(57.) How many bushels of wheat, at $1,50 per bushel, must be given in exchange for 84 yards of cloth, at $1,25 per yard? Ans. 70 bushels.

(58.) At £$1\frac{1}{2}$ per cwt., what will $3\frac{1}{3}$lb. cost?
Ans. $10\frac{5}{7}$d.

(59.) If $4\frac{1}{2}$ cwt. can be carried 36 miles for 35 shillings, how many pounds can be carried 20 miles for the same money? Ans. $907\frac{1}{5}$lb.

(60.) What must I allow my correspondent for selling goods to the amount of $2317,46, at a commission of $3\frac{1}{4}$ per cent.? Ans. $75,317.

(61.) If $\frac{5}{8}$ of a yard cost £$\frac{4}{7}$, what will $\frac{9}{13}$ of an ell English cost? Ans. 17s. 1d. $2\frac{4}{7}$qr.

(62.) What decimal is equivalent to $\frac{8}{11}$?
Ans. ,7272+.

(63.) What part of a day is 3 hours, 14 minutes, 24 seconds? Ans. $\frac{27}{200}$ day.

(64.) What is the interest of $560 for 10 months, at 6 per cent.? Ans. $28.

(65.) If 12 oz. of wool make $1\frac{1}{2}$ yard of cloth $\frac{7}{8}$ wide, how many $1\frac{1}{4}$ yard wide, will 16 lb. of wool make?
Ans. $22\frac{2}{5}$ yards.

(66.) What is the side of a square block of marble that shall contain 13824 cubic inches? Ans. 2 feet.

(67.) Suppose a square floor to contain 20736 square inches; how many feet long is one side? Ans. 12 feet.

(68.) A guardian paid his ward $3500 for $2500 which he had held in his possession 8 years; what rate of interest did he allow him? Ans. 5 per cent.

(69.) What is the compound interest of $50, for 3 years, at 5 per cent.? Ans. $7,88+.

(70.) What is the amount of $63, for 2 years, at 6 per cent. compound interest? Ans. $70,786+.

(71.) If a man having bought 359 yards of cloth for $621, at what must he sell them, per yard, to make 15 per cent upon the sale? Ans. $1,989+.

(72.) In what time will $627,50, loaned at 7 per cent., produce as much interest as $2510, at $3\frac{1}{2}$ per cent., will produce in 1 year and 8 months? Ans. $3\frac{1}{3}$ years.

(73.) Sold goods to the amount of $1200, on 6 months' credit; what is the present worth, allowing 8 per cent. discount? Ans. $1153,846+.

(74.) Sold 48 cwt. of hops at 4d. per pound, and received pay in prunes at 6d. per pound; how many cwt. did I receive? Ans. 32 cwt.

(75.) What length must be cut off from a board, $8\frac{3}{8}$ inches wide, to contain a square foot? Ans. $17\frac{13}{67}$in.

(76.) What is the amount of \$500 for 3 years, at 5 per cent. compound interest? Ans. \$578,812+.

(77.) Bought goods amounting to \$615,75, at 6 months' credit; how much ready money must be paid, if a discount of $4\frac{1}{2}$ per cent. be allowed? Ans. \$602,20+.

(78.) A cistern containing 60 gallons of water, has 3 unequal cocks for discharging it; the greatest cock will empty it in one hour; the second in two hours; the third in 3; in what time will it be emptied, if they all run together? Ans. $32\frac{8}{11}$ minutes.

(79.) A can do a piece of work in 10 days, and B in 13; in what time can they do it, both working together? Ans. $5\frac{15}{23}$ days.

(80.) There is a prize of £212 14s. 7d. to be divided amongst a certain captain, 4 men and a boy; the captain is to have a share and a half; the men each a share; and the boy $\frac{1}{3}$ of a share; what ought each person to have?

Ans. The captain £54 14s. $\frac{3}{7}$d.; each man £36 9s. $4\frac{2}{7}$d.; and the boy £12 3s. $1\frac{3}{7}$d.

(81.) There is an island 73 miles in circumference, and 3 footmen all start to travel together the same way about it; A goes 5 miles a day; B 8 and C 10; when will they all come together again? Ans. 73 days.

(82.) What is the cost of a lot of land 62ft. $11\frac{3}{4}$in. long, and 27ft. $3\frac{1}{2}$in. wide, at \$1,80 per square foot? Ans. \$3093,86.

(83.) Suppose \$984,37½ was paid in New Orleans for a draft on New York, when the advance was 5 per cent; what was the draft drawn for? Ans. \$937,50

(84.) In a thunder storm, 6 minutes elapsed between the lightning and the thunder; at what distance was the explosion—sound moving at the rate of 1142 feet in a second? Ans. 6852 ft.$=1\frac{131}{440}$ miles.

(85.) What number is that to which if $\frac{2}{7}$ of $\frac{5}{8}$ be added, the sum will be 1? Ans. $\frac{23}{28}$.

(86.) Divide \$1000; and give A \$120 and B \$195 less than C. Ans. A \$445; B \$230; C \$325.

ERRATA.

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§ 40. 4. Page 53, (1.) Ans. should be 183$\frac{[illegible]}{252}$.
§ 41. 1. " " Case should be V.
§ 42. 1. " 55, " " " VI.
§ 79. 1. " 85, (10.) Ans. should be $\frac{559}{432}$.
§ 84. 2. " 89, Expressing should be, instead of expresing.
" 6. " " Vulgar " " vulger.
" 7. " " Last word should be Separatrix, instead of separating.
§ 100. 8. Page 106, (20.) Should be, From \$125, instead of from \$1,25.
§ 102. V. Page 113, In first line, after dry measure, should be inserted, custom house measurement.
At the close of the first paragraph there should be—But in New York, according to the Revised Laws, the bushel contains 2211,84; and the dry gallon 276,48 cubic inches.
VI. Page 113, In the last paragraph, after wine measure, should be inserted, custom house measurement; instead of 8lbs. should be 8,339lbs.; and at its close should be—But in New York, the wine gallon holds 8lbs. and contains 221,18 cubic inches.
§ 104. 8. Page 123, (1.) Ans. should be $\frac{960}{5}$.
§ 113. 3. " 142, (17.) Ans. " " £21 9s. 5d. 3qr.
§ 125. 2. " 162, (3.) Ans. " " £2 0s. 2$\frac{2}{9}$d.
" 5. " 163, (12.) Ans. " " £155 13s.
" " " " (19.) Ans. " " £75 17s. 3$\frac{3}{16}$.
" " " " (23.) Ans. " " £292 2s. 10$\frac{2}{7}$d.
" " " 164, (34.) Ans. " " £1428 15s. 2d.
§ 126. " 165, (9.) Ans. " " $\frac{25}{567}$ bar.
" " 167, (29.) Ans. " " \$18,535+.
§ 129. 4. " 176, (21.) Ans. " " 11$\frac{1}{5}$ days.
" " " 179, (64.) Should be omitted.
§ 137. 1. " 191, Note; should be added to the note—But for the greatest accuracy, the year should be called 365 days.
§ 139. 2. Page 192, The comparison should be \$$\frac{100}{50}$ of $\frac{1}{2}$yr. of \$7.

The above corrections can most of them be quickly and easily made by either the teacher or pupil with a fine pencil.